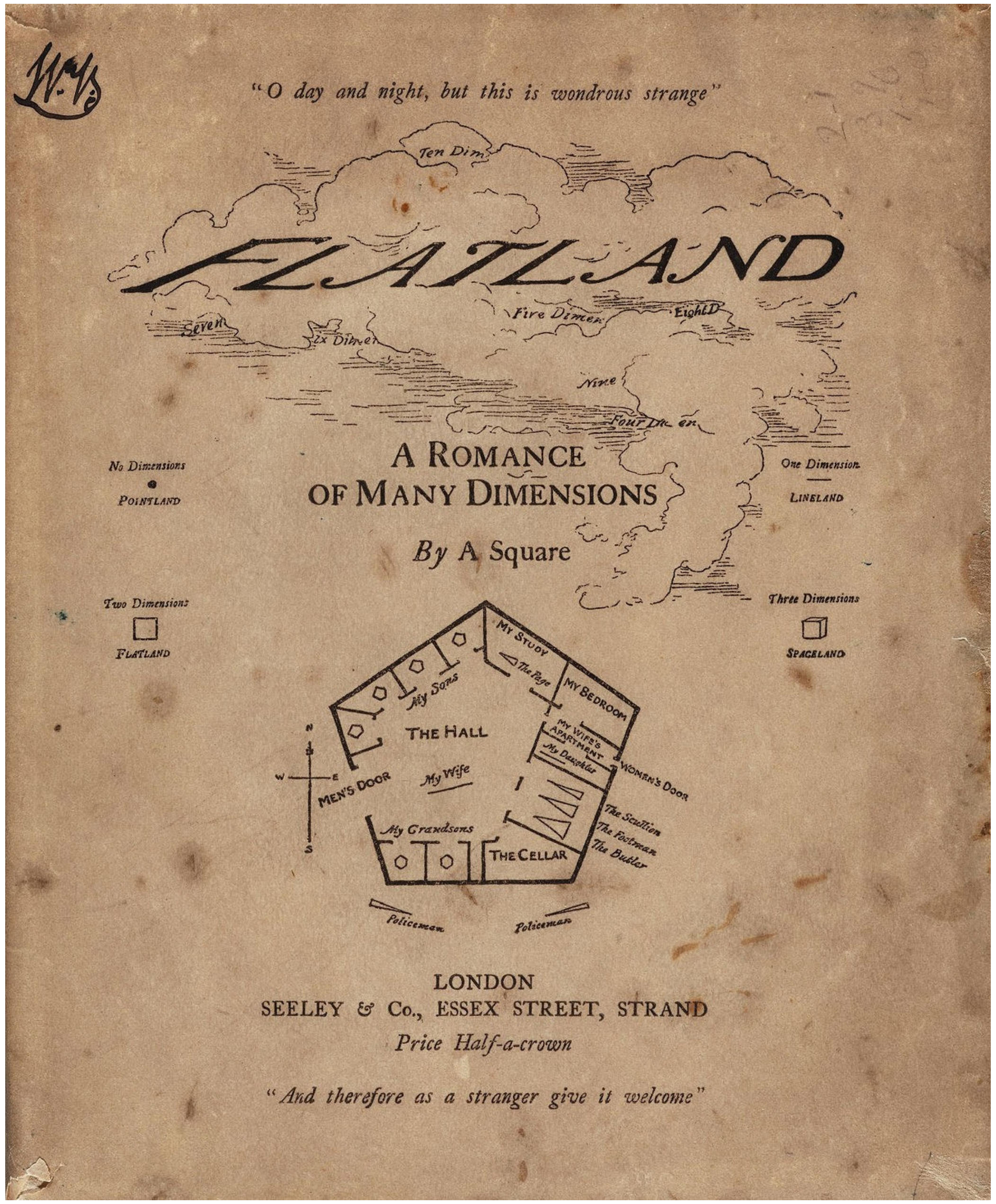
Superfluids in Flatland

Topology, Defects, and the 2016 Nobel Prize in Physics



Sid Parameswaran

Saturday Mornings of Theoretical Physics
Oxford, October 28, 2017

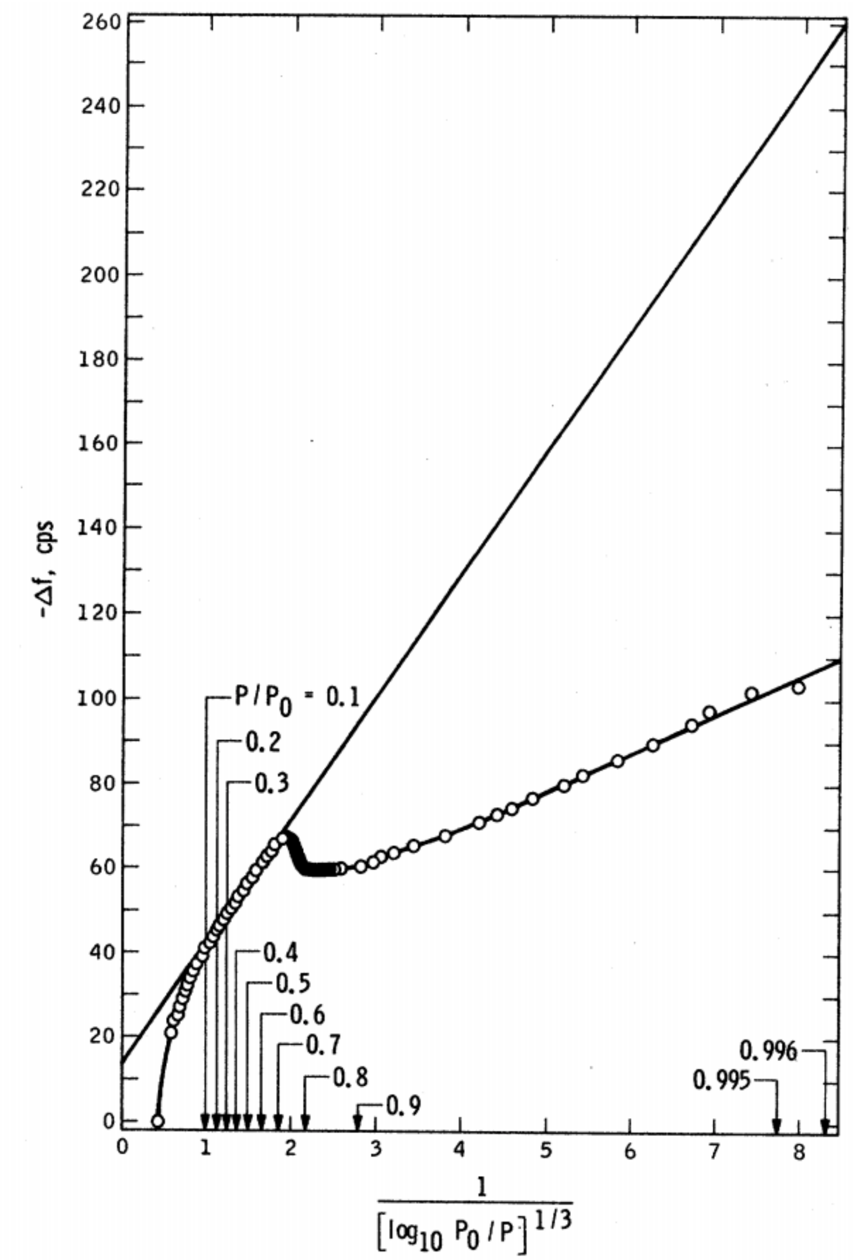
Image Credit: Wikipedia

Superfluids in Flatland

By the early 70s, experimentalists were able to create "thin films" of helium moving on some substrate.

Experiments measured **deviations in the mass of helium "stuck" to the substrate**, indicative of superfluidity.
(some of the helium has frictionless flow!)

This contradicts conventional theoretical wisdom that superfluids can't exist in 2D.



This conventional wisdom, why it needs to be amended, and the eventual explanation of 2D superfluidity, are the subjects of this talk.

[M. Chester, L. C. Yang, and J. B. Stephens Phys. Rev. Lett. 29, 211 (1972)]

Sure it works in *practice*, but does it work in theory?

Superfluids

Macroscopic wave function $\Psi(\mathbf{r}) = |\Psi(\mathbf{r})|e^{i\varphi(\mathbf{r})}$

Superfluid Mass Density $\rho_s(\mathbf{r}) = m|\Psi(\mathbf{r})|^2$

Superfluid Velocity $\mathbf{v}_s = \frac{\hbar}{m}\boldsymbol{\nabla}\varphi(\mathbf{r})$

Kinetic energy per unit area: $\frac{1}{2}\rho_s(\mathbf{r})\mathbf{v}_s(\mathbf{r})^2$

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Kinetic energy per unit area: $\frac{1}{2}\rho_s(\mathbf{r})\mathbf{v}_s(\mathbf{r})^2$

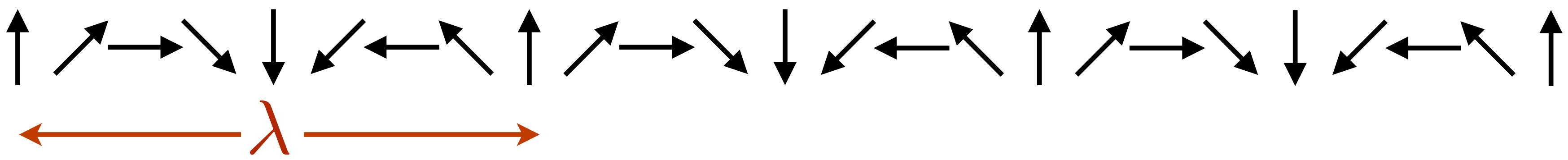
Simplification: assume superfluid density is (roughly) constant everywhere

$$E \approx \frac{\hbar^2 \rho_s}{2m^2} \int d^2x \, (\boldsymbol{\nabla}\varphi)^2$$

"Phase Waves"

$$E \propto \int d^2x \, (\boldsymbol{\nabla}\varphi)^2$$

gradual spatial "twist" of phase
costs very little energy



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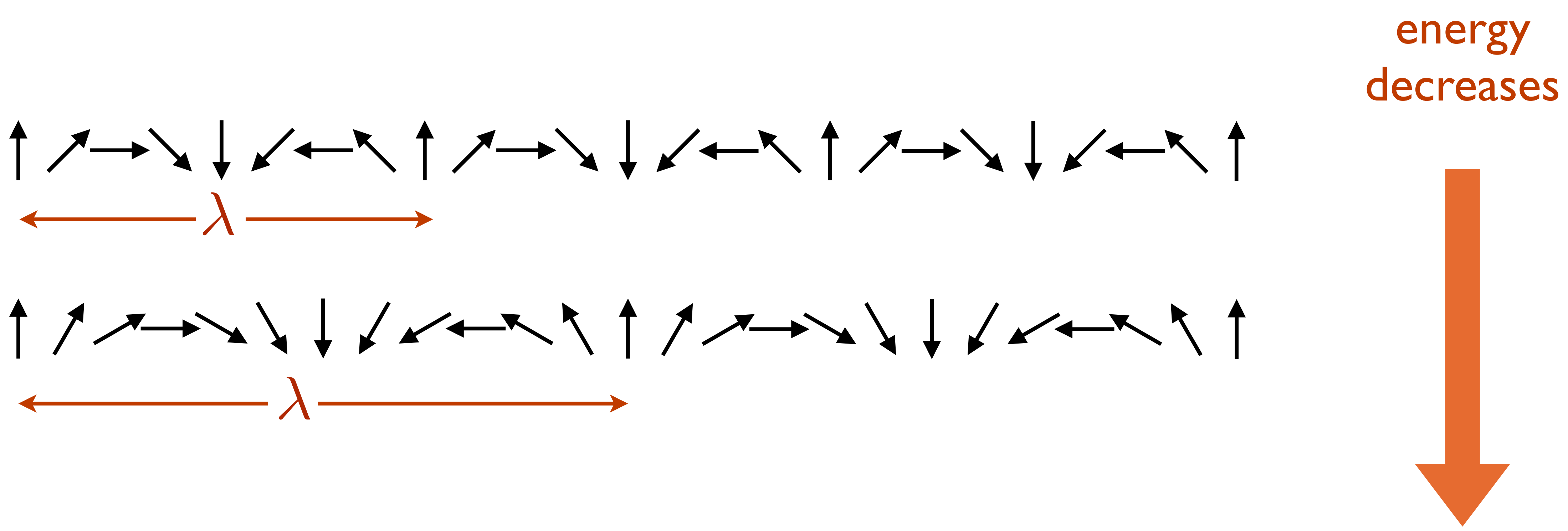

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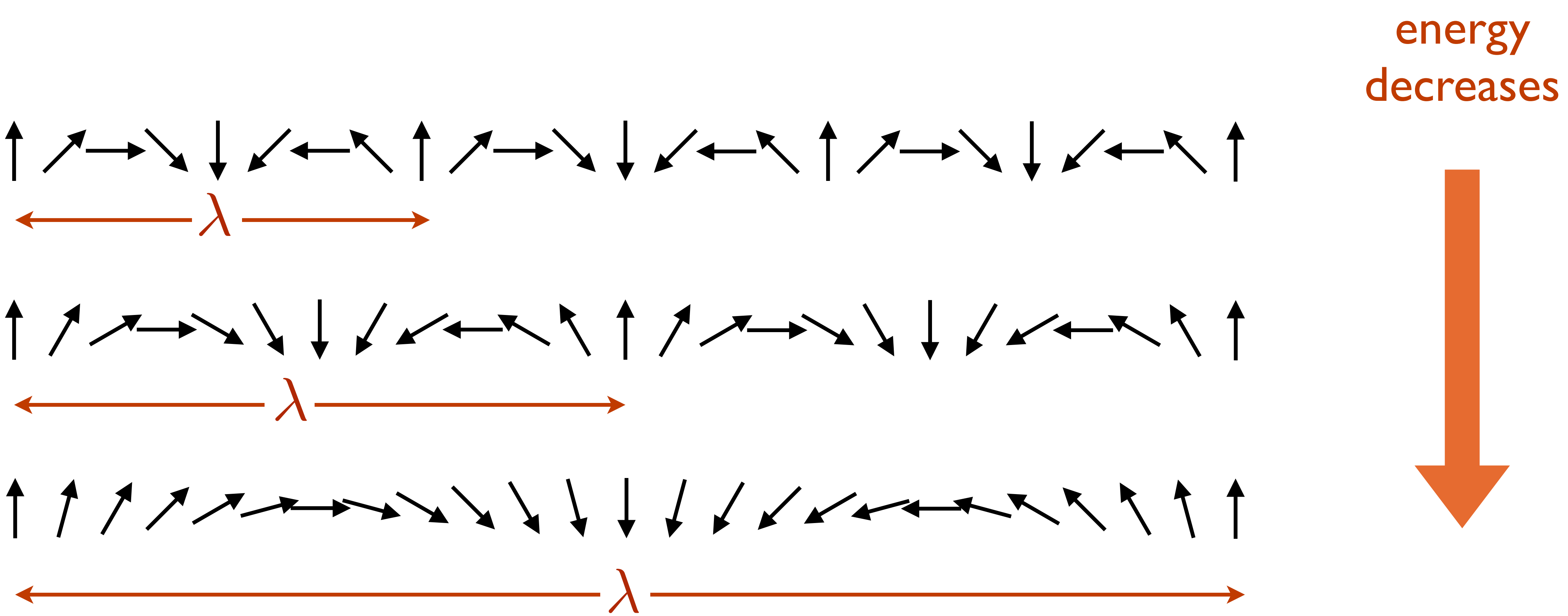

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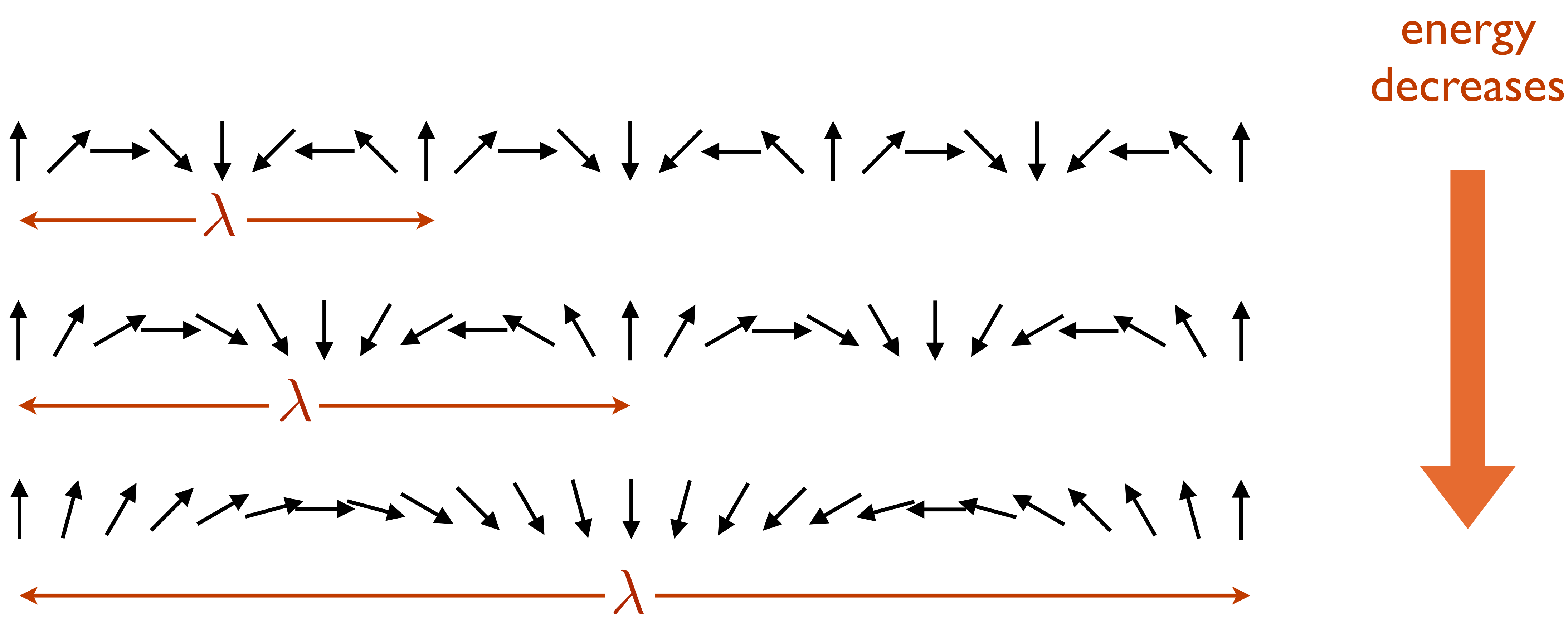
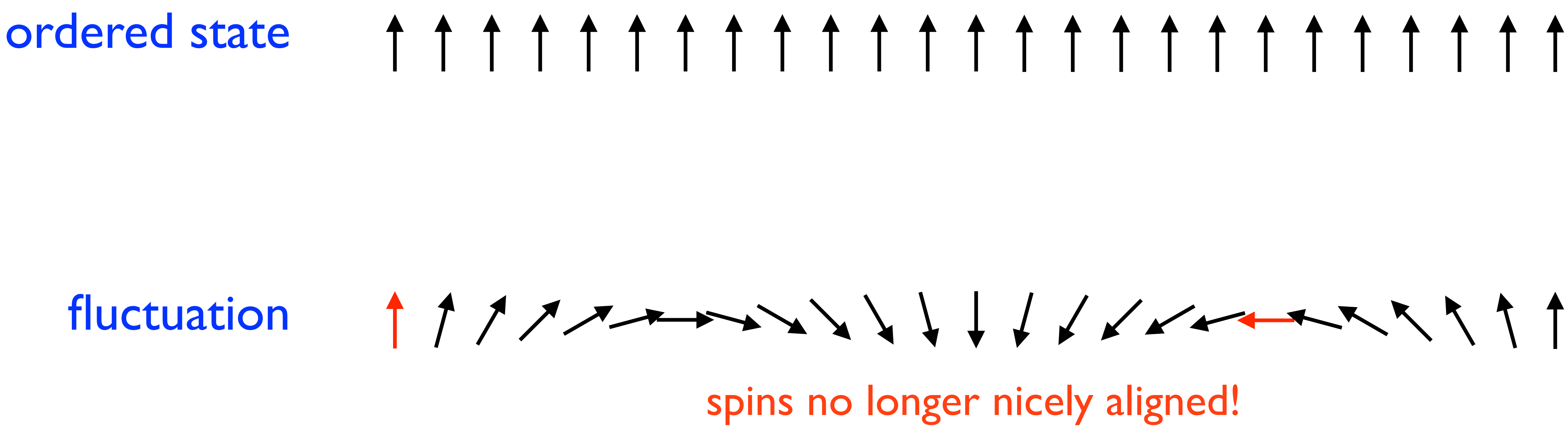


(This is an example of a "Goldstone mode", required by general theorems)

<u>Similar examples</u>: sound waves in solids, spin waves in magnets,…

Fluctuations

Phase-wave modes allow low-energy **fluctuations** away from "perfect" order



Fluctuations

Statistical Mechanics:

probability of fluctuations with energy E at temperature T $\quad \propto e^{-\frac{E}{k_B T}}$

(so, low-energy fluctuations like phase waves are very important)

Fluctuations

Statistical Mechanics:

probability of fluctuations with energy *E* at temperature *T* $\propto e^{-\frac{E}{k_B T}}$

(so, low-energy fluctuations like phase waves are very important)

Such fluctuations can destroy order (usually happens in 1D/2D)

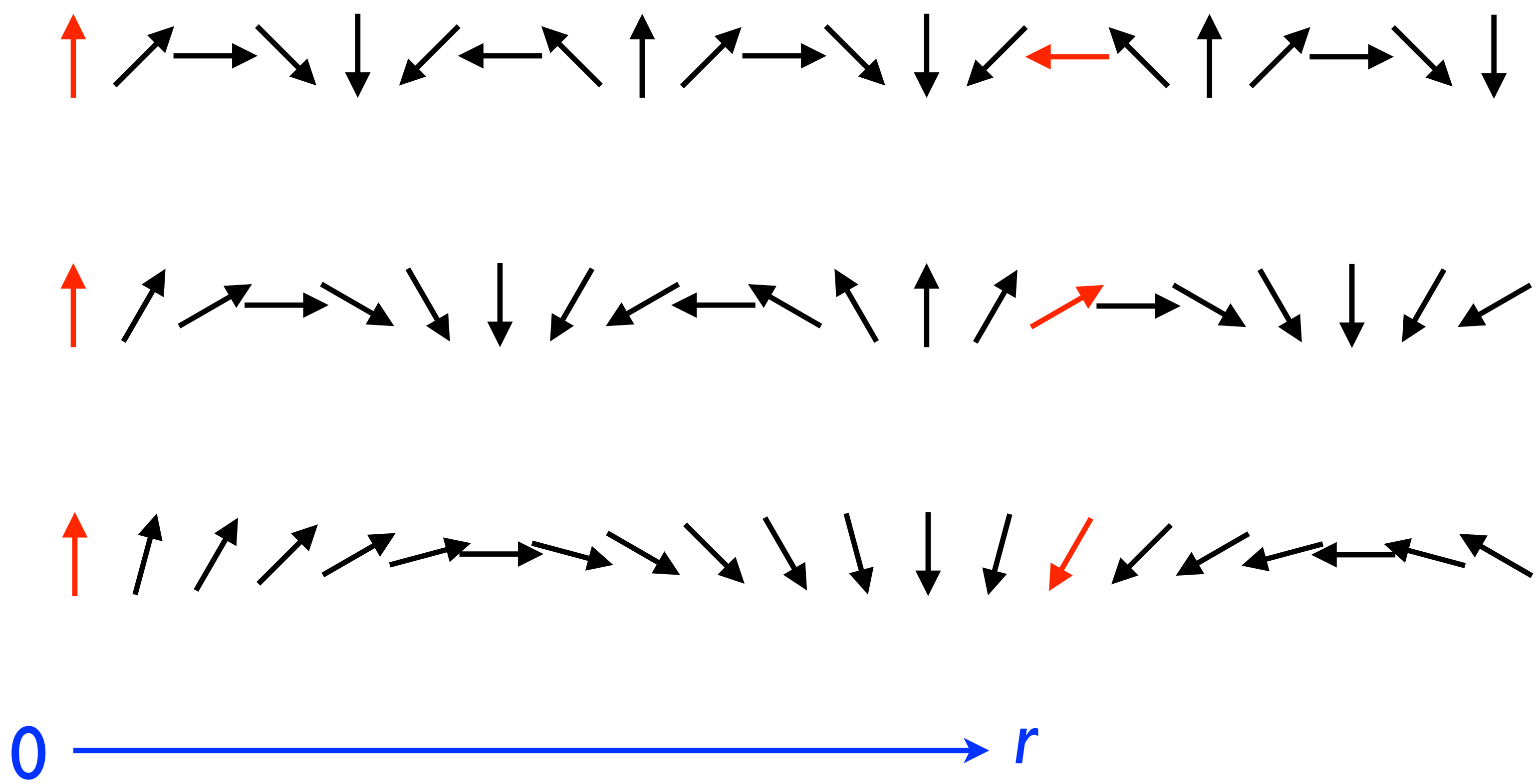
[Variously attributed to Hohenberg/Mermin/Wagner/Coleman (c. 1950s), but originally noted by Peierls for 2D solids in 1934!]

Fluctuations in 2D Superfluids

$$E \propto \int d^2x \, (\nabla\varphi)^2$$

probability $\propto e^{-\frac{E}{k_B T}}$

Phase-wave modes of many <u>different</u> wavelengths can be thermally excited:



What's the phase angle between **0**, ***r*** ?

Fluctuations in 2D Superfluids

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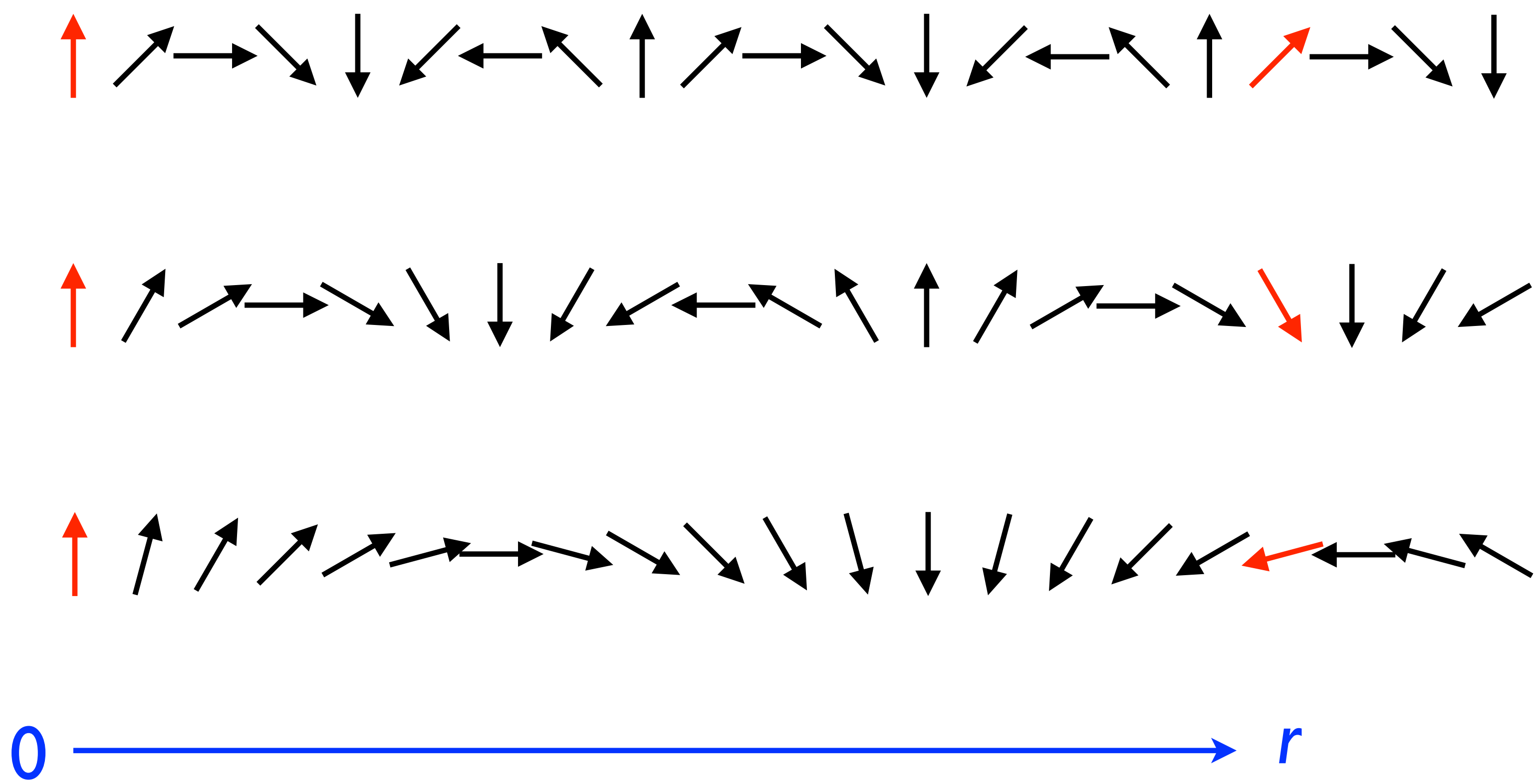

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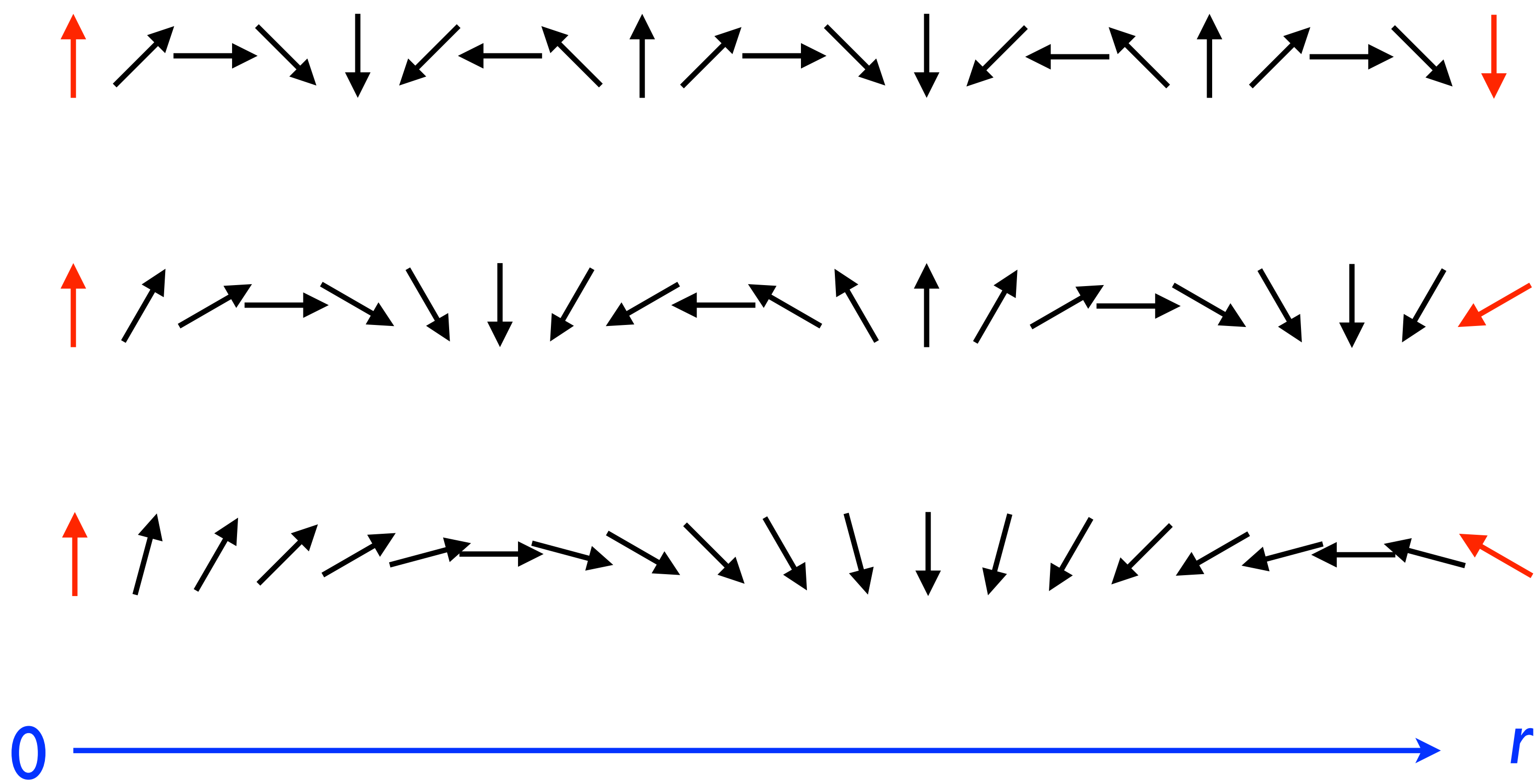

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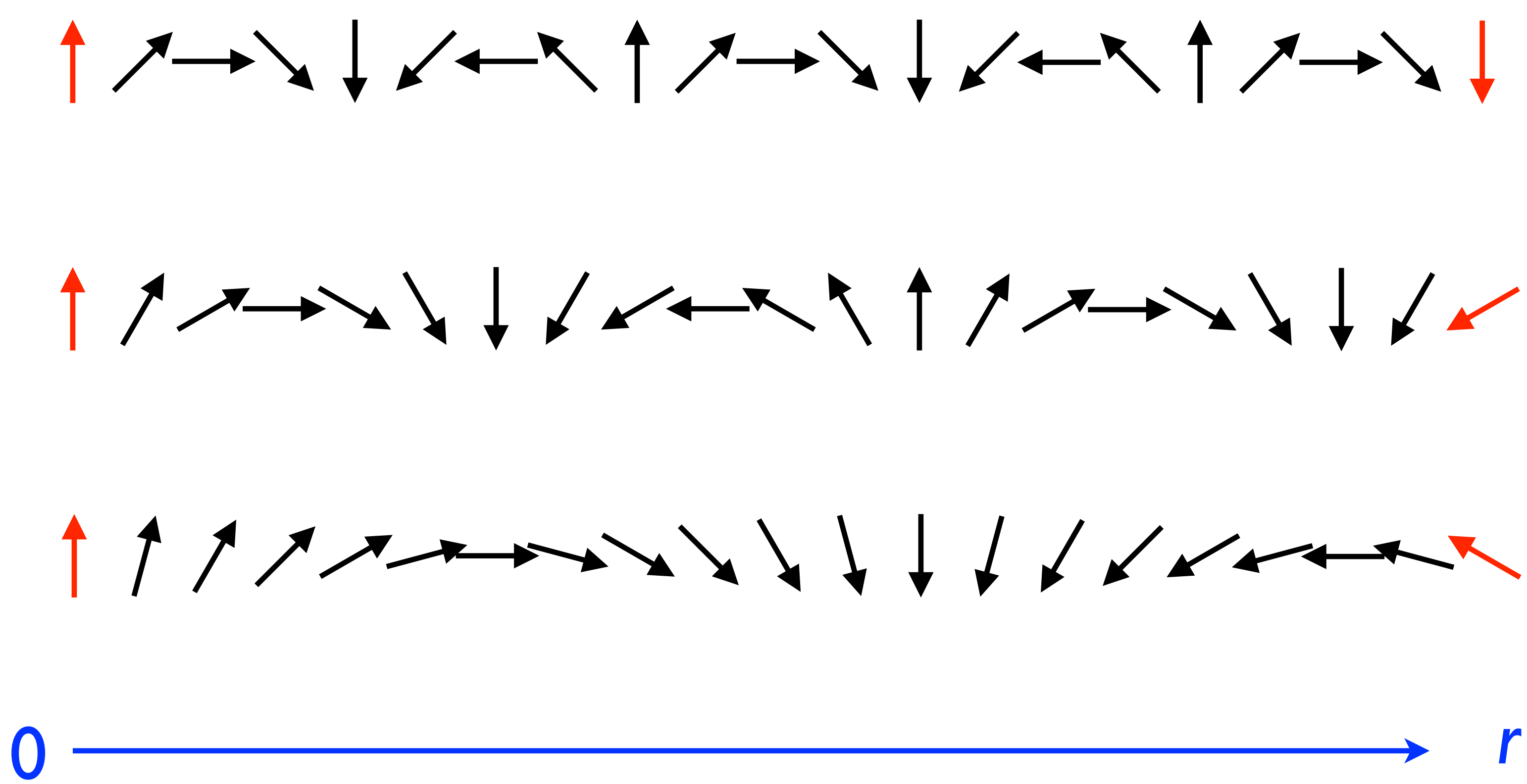


What's the phase angle between **0**, *r* ?

Averaging over possibilities (statistical mechanics) gives

$$\langle \cos(\varphi_{\mathbf{r}} - \varphi_{\mathbf{0}}) \rangle \propto \left(\frac{a}{r}\right)^{\zeta}$$

no "long range order": phases at **0**, *r* not "coherent" as $r \rightarrow \infty$

Logical Conclusion:

Long-range phase order is impossible, so 2D superfluids shouldn't exist.

But… a Puzzle

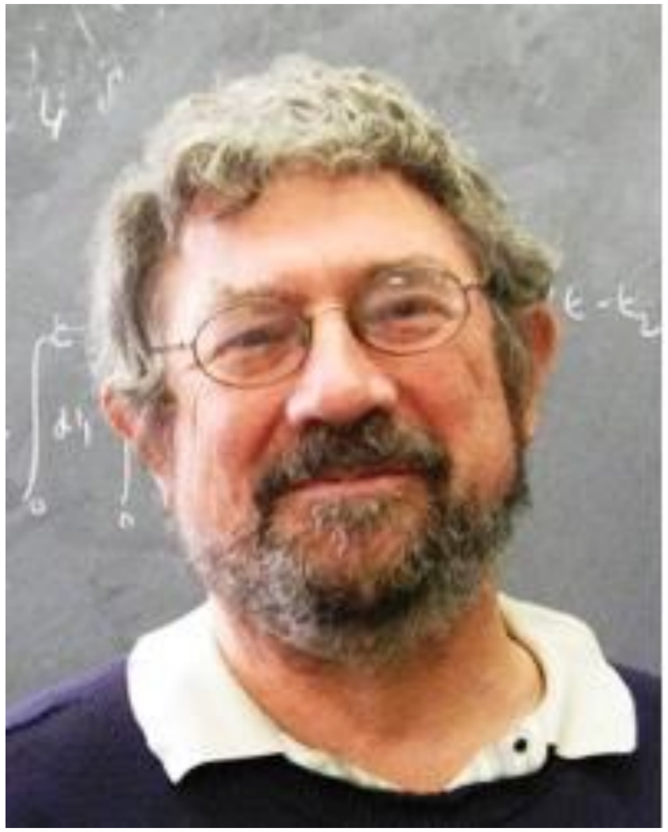
At low temperatures, $\langle \cos(\varphi_{\mathbf{r}} - \varphi_{\mathbf{0}}) \rangle \propto \left(\frac{a}{r}\right)^{\zeta}$

But, at *high* temperatures, $\langle \cos(\varphi_{\mathbf{r}} - \varphi_{\mathbf{0}}) \rangle \propto e^{-r/\xi}$

High-*T* and low-*T* don't match, <u>but</u> no ordering occurs.

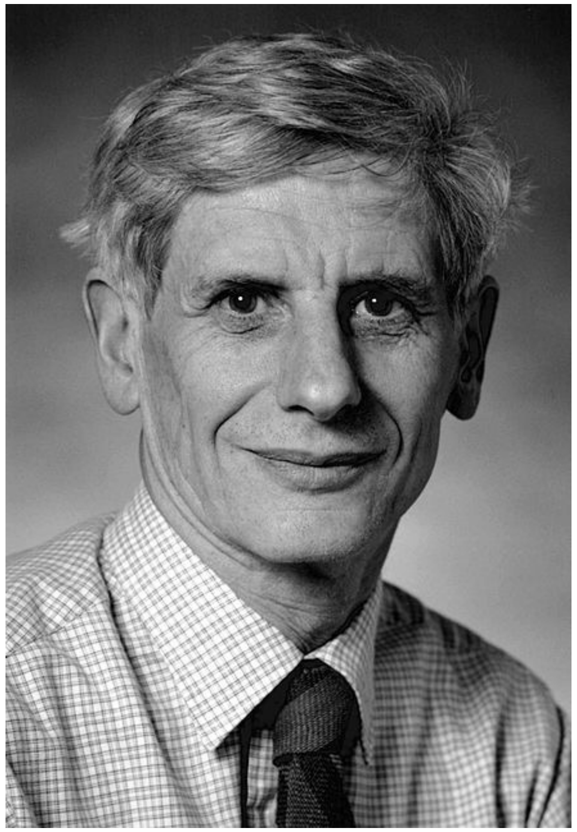
What's going on?

The answer was provided by



J. M. Kosterlitz

(Oxford D. Phil 1966-69)



D. J. Thouless

(work described here was done at Birmingham in the 1970s)

Image Credits: Brown University; Mary Levin/University of Washington

Aside: In Praise of Procrastination

After his D. Phil — in particle physics — Kosterlitz took a short-term post-doc in Torino.

When this ended, he wanted to move to CERN, where all the excitement was.

However, he failed to submit his application on time, and so instead ended up in Birmingham for another post-doc.

"I had been doing long tedious calculations for very little return, and I was getting a bit fed up. So I started walking from office to office asking if anybody had a tractable problem I could work on. I found myself in David Thouless's office where he was telling me all about superfluid helium films, crystals, dislocations, vortices — all concepts that were completely new to me. But somehow what he was saying made sense. So I started working on some of the ideas he was throwing out, and basically things worked out."

— J.M. Kosterlitz

Back to our simplified model

$$E \propto \int d^2x \, (\boldsymbol{\nabla} \varphi)^2$$

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Missing a crucial ingredient:
vortices

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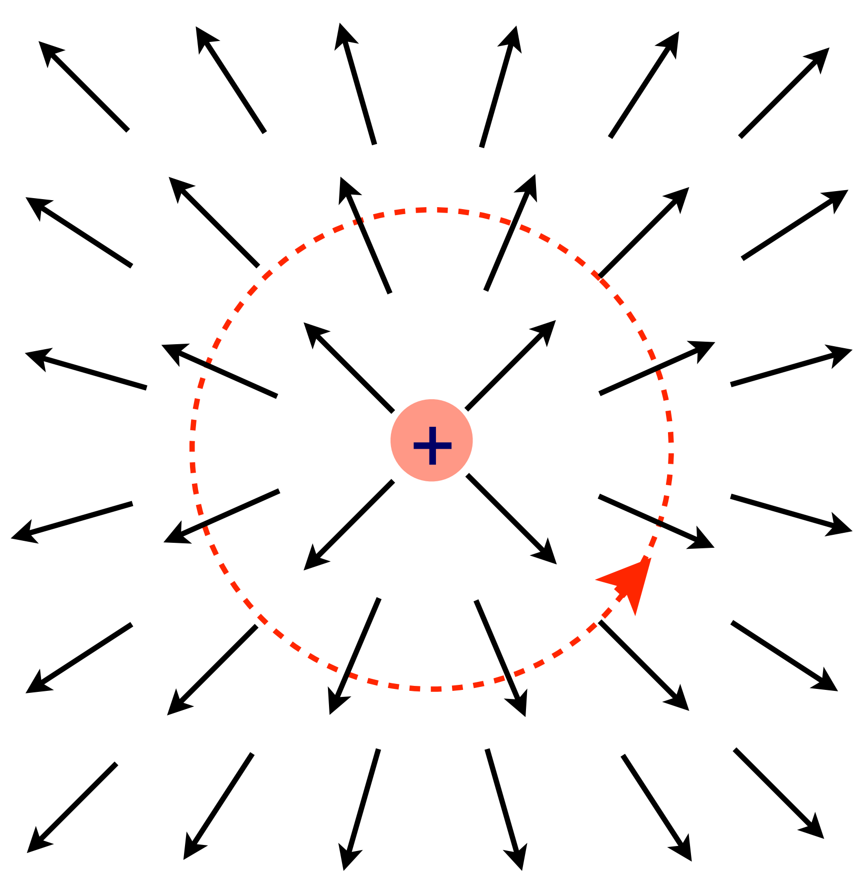
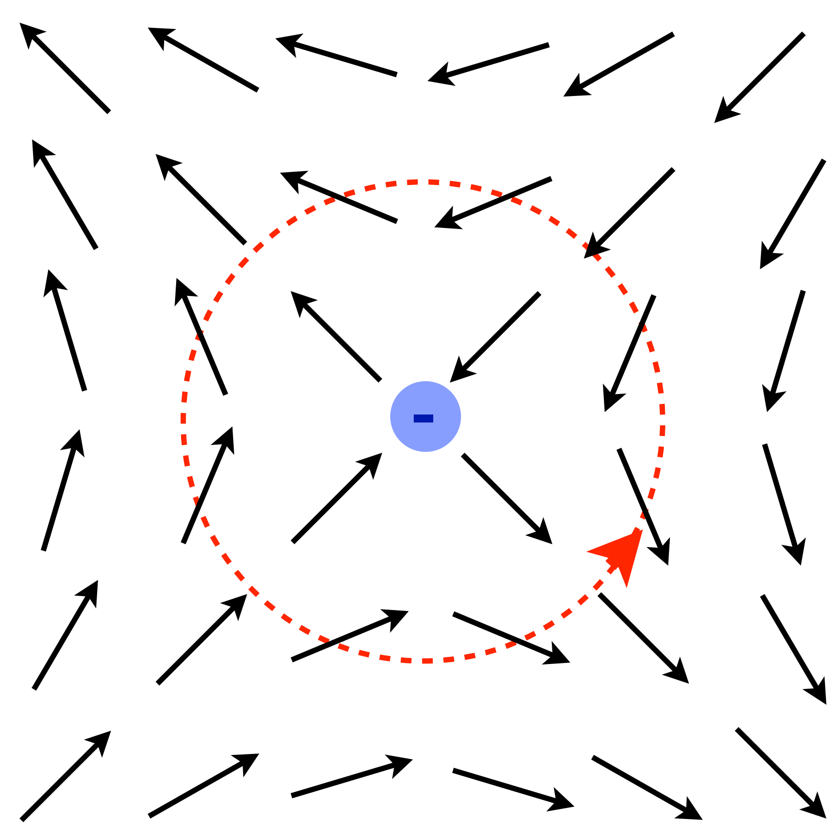
Missing a crucial ingredient:
vortices

Vortex: point around which $\boldsymbol{\varphi}$ winds by 2π×(integer)

$$\oint \boldsymbol{\nabla}\varphi(\mathbf{r}) \cdot d\mathbf{r} = 2\pi \times n$$



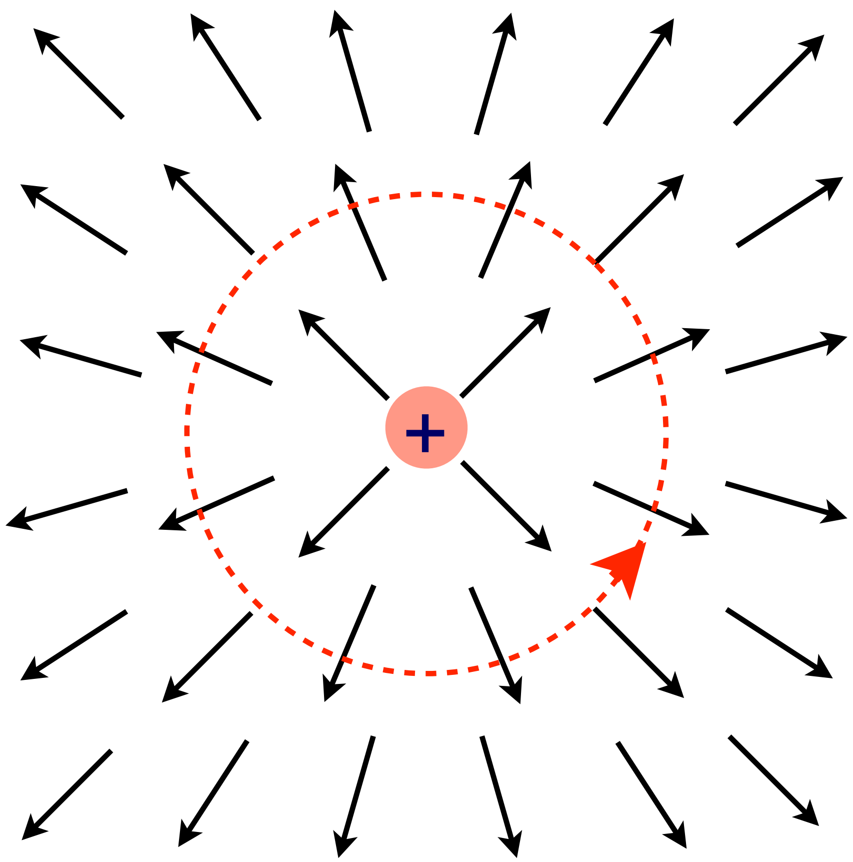
single vortex $(n = 1)$

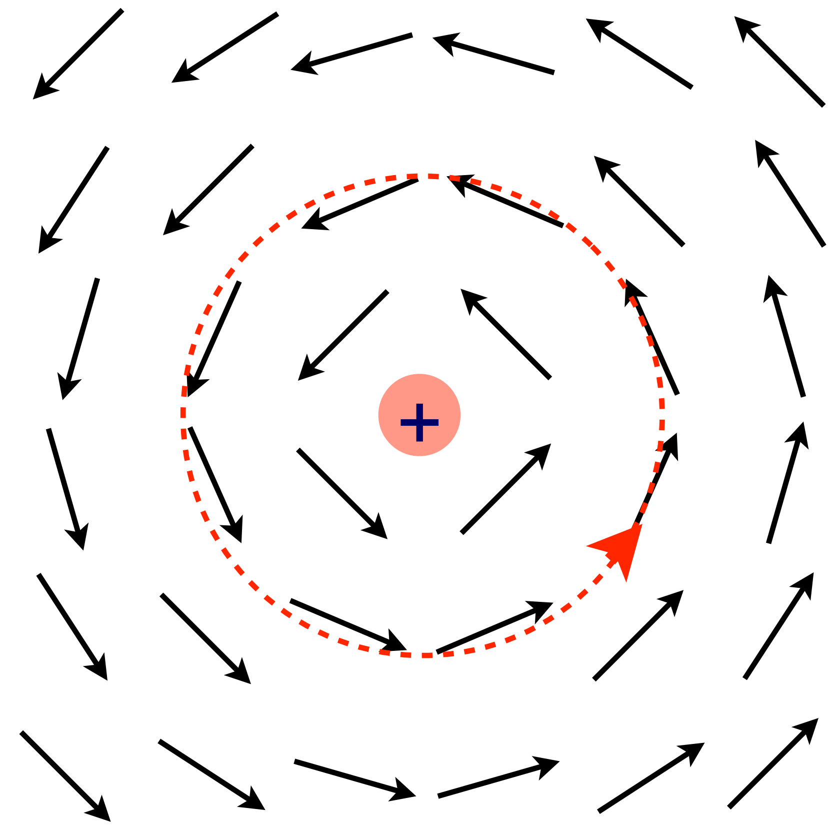


single anti-vortex $(n = -1)$

Vortices are topologically stable

$$\oint \nabla \varphi(\mathbf{r}) \cdot d\mathbf{r} = 2\pi$$





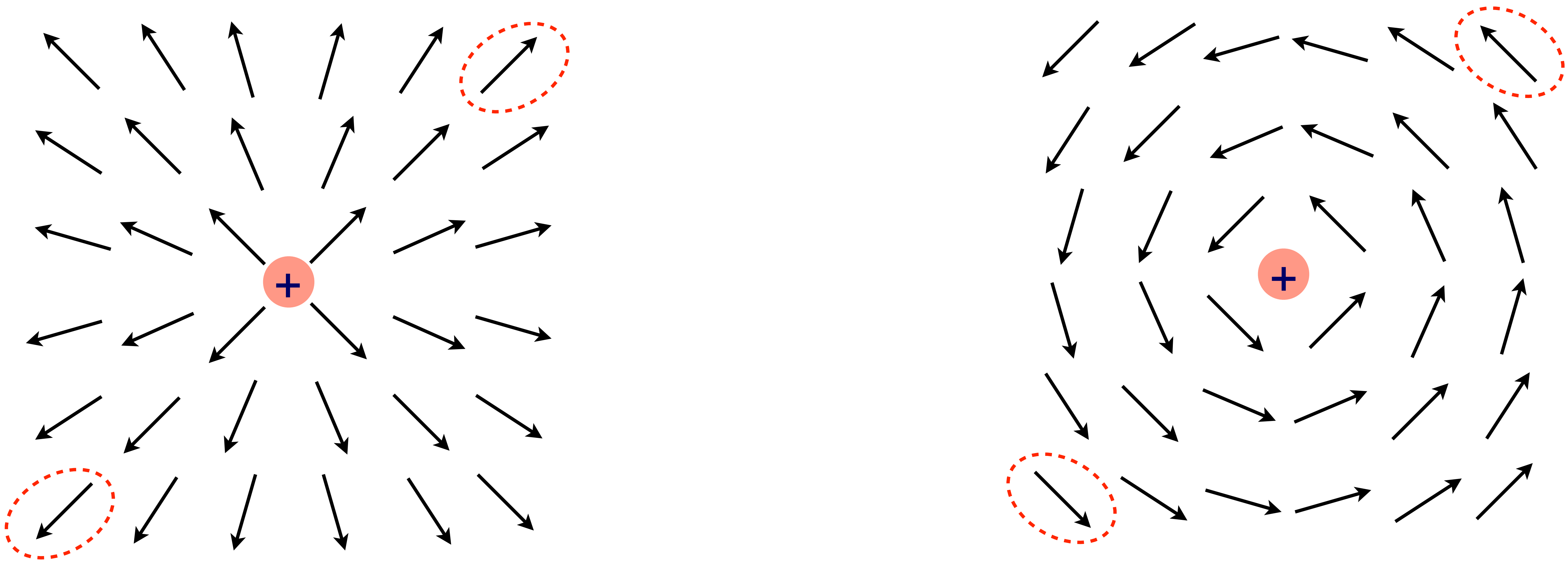
No smooth twist (e.g. like those in a phase-wave)
can change the total phase winding around the circle.

The quantized "charge" (winding) is a topological invariant.

[recall the quantized vorticity in John Chalker's talk]

Vortices can destroy long-range order

phases around a vortex can't be aligned



(no matter how you try to smoothly rotate them,
some arrows will always be misaligned)

This is linked to their topological stability.

Better Description of Superfluid

$$E \propto \int d^2x \, (\boldsymbol{\nabla}\varphi)^2$$

(phase waves)

+ terms describing vortices

We make "simplifying approximations" all the time by ignoring terms.

When is it important to keep the vortices?

To answer: need to understand if vortices can cause transitions.

Phase Transitions

Since superfluid transition is driven by temperature *T*, we need to look at *free energy*

$$F = E - TS$$

E: energy
S: entropy

Thermodynamics: systems seek to minimize *F*

Low temperatures: lowest-energy state wins

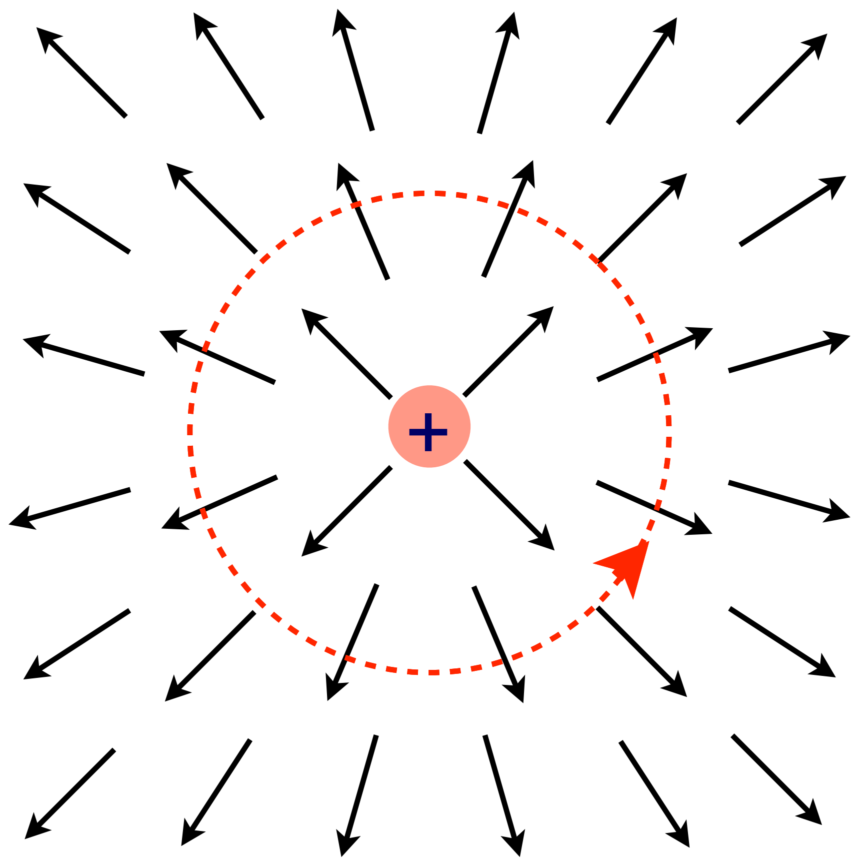
(e.g. crystalline solid)

High temperatures: high-entropy state wins

(e.g. gas)

So we need to calculate *E*, *S* for vortices

Step 1: Vortex Energy



Quantized vorticity: $$\oint \boldsymbol{\nabla}\varphi(\mathbf{r}) \cdot d\mathbf{r} = 2\pi$$

$$2\pi r(\boldsymbol{\nabla}\varphi) \sim 2\pi$$

$$\boldsymbol{\nabla}\varphi(\mathbf{r}) \sim \frac{1}{r}$$

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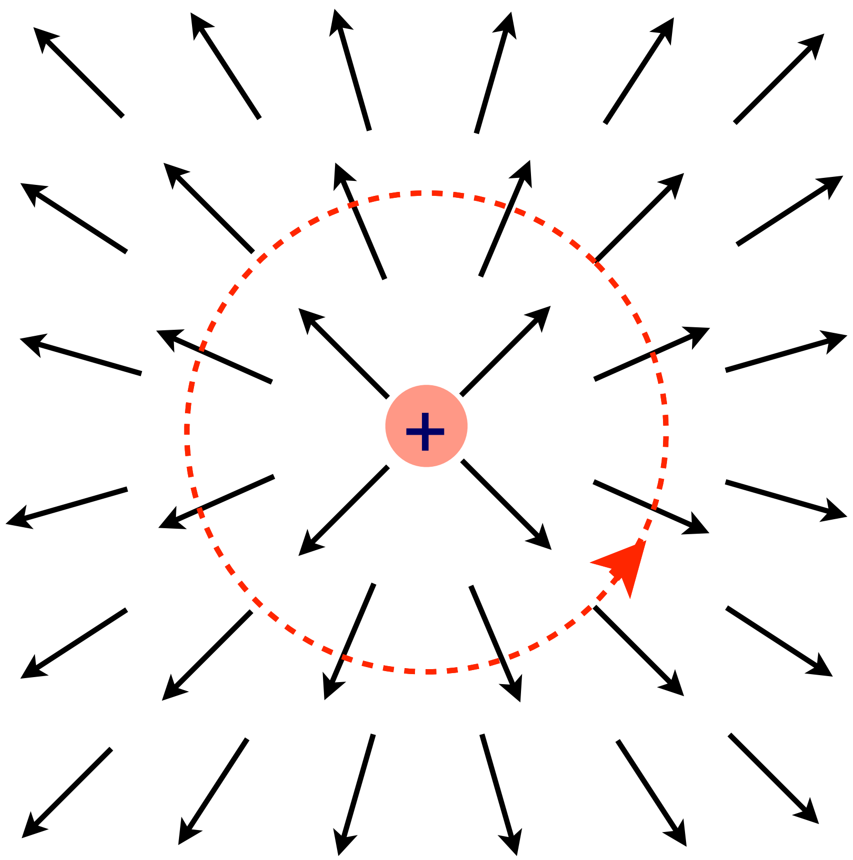

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<u>Vortex energy:</u> cost of this "phase twist"

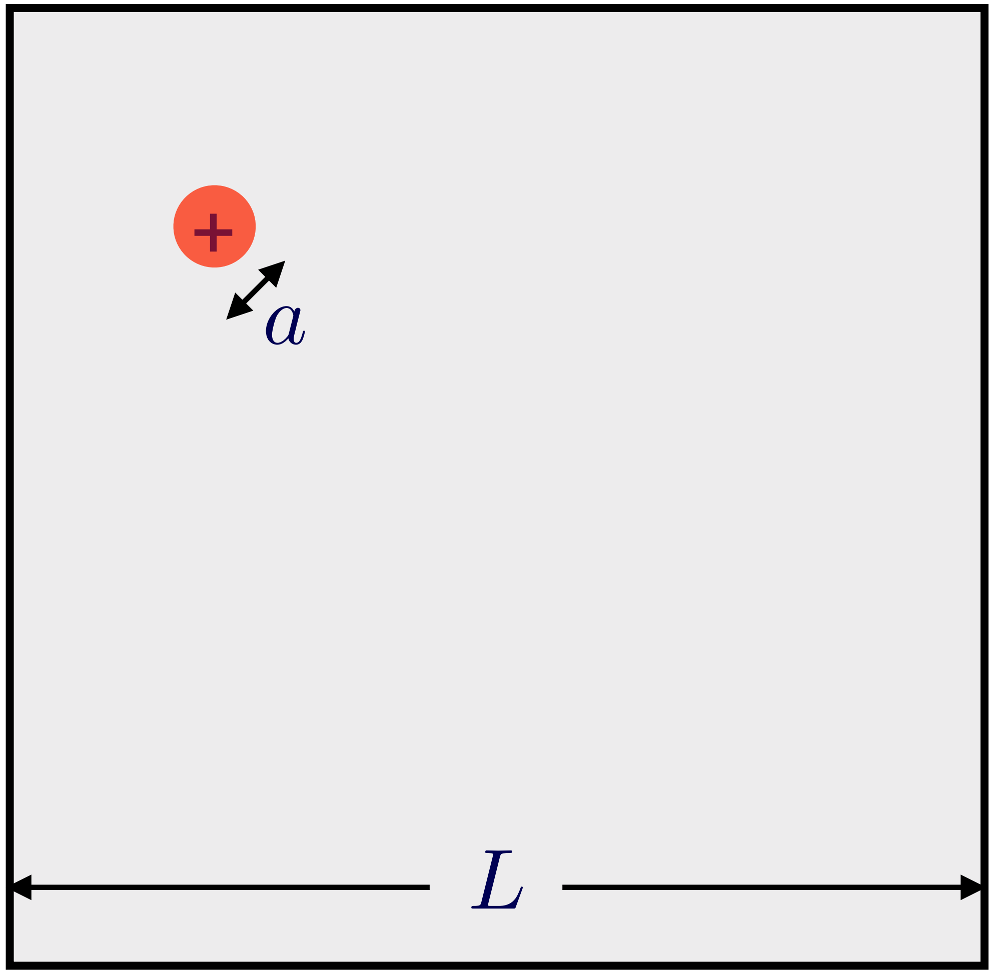
$$E_v \sim \int d^2x \, (\boldsymbol{\nabla}\varphi)^2 \sim \int \frac{d^2x}{r^2} \propto \ln \frac{L}{a}$$

a: size of vortex core
L: size of system

Step 2: Vortex Entropy

Entropy ~ ln (number of configurations)

Different vortex configurations $\longleftrightarrow$ different core location



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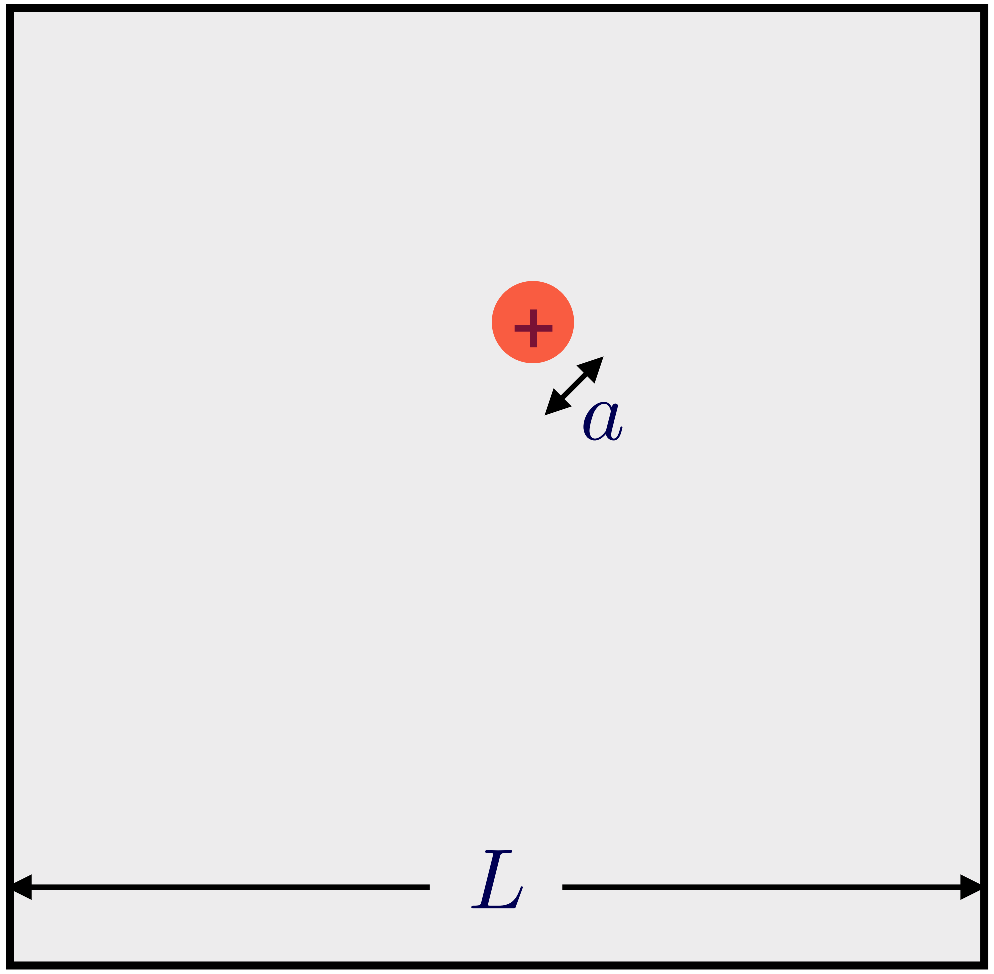

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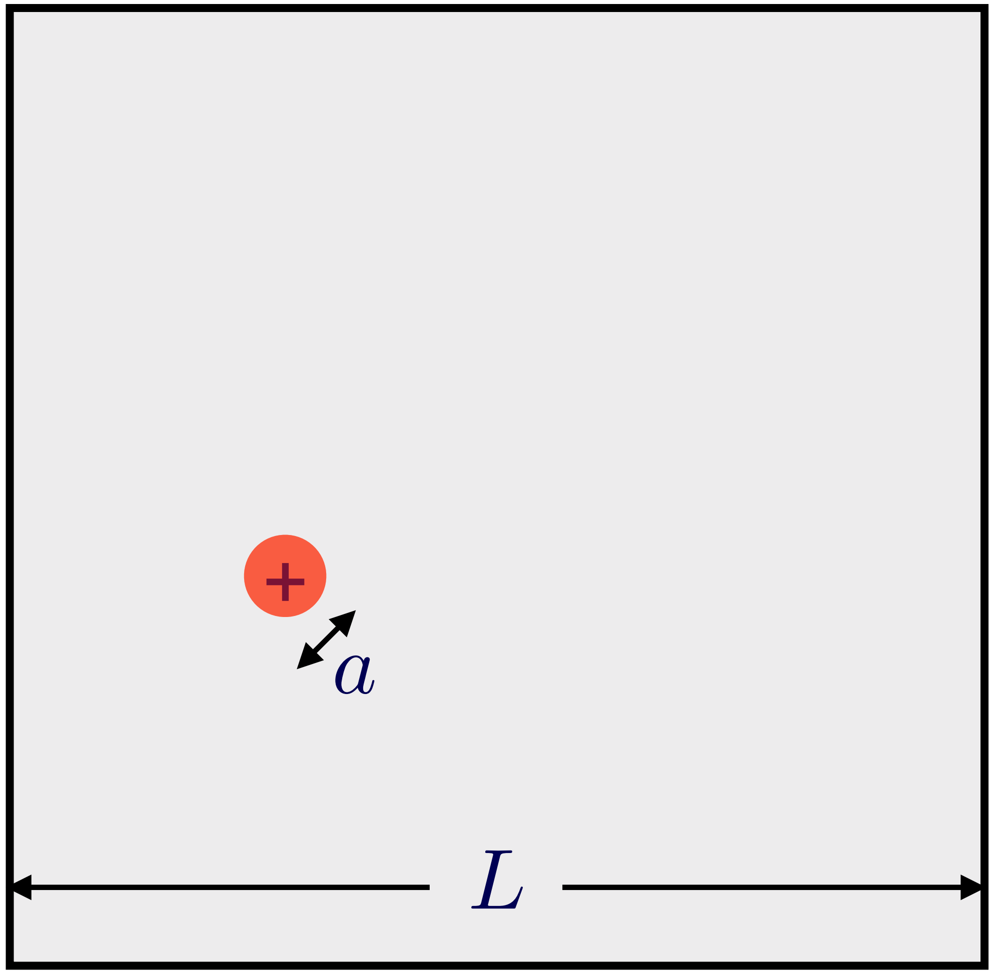

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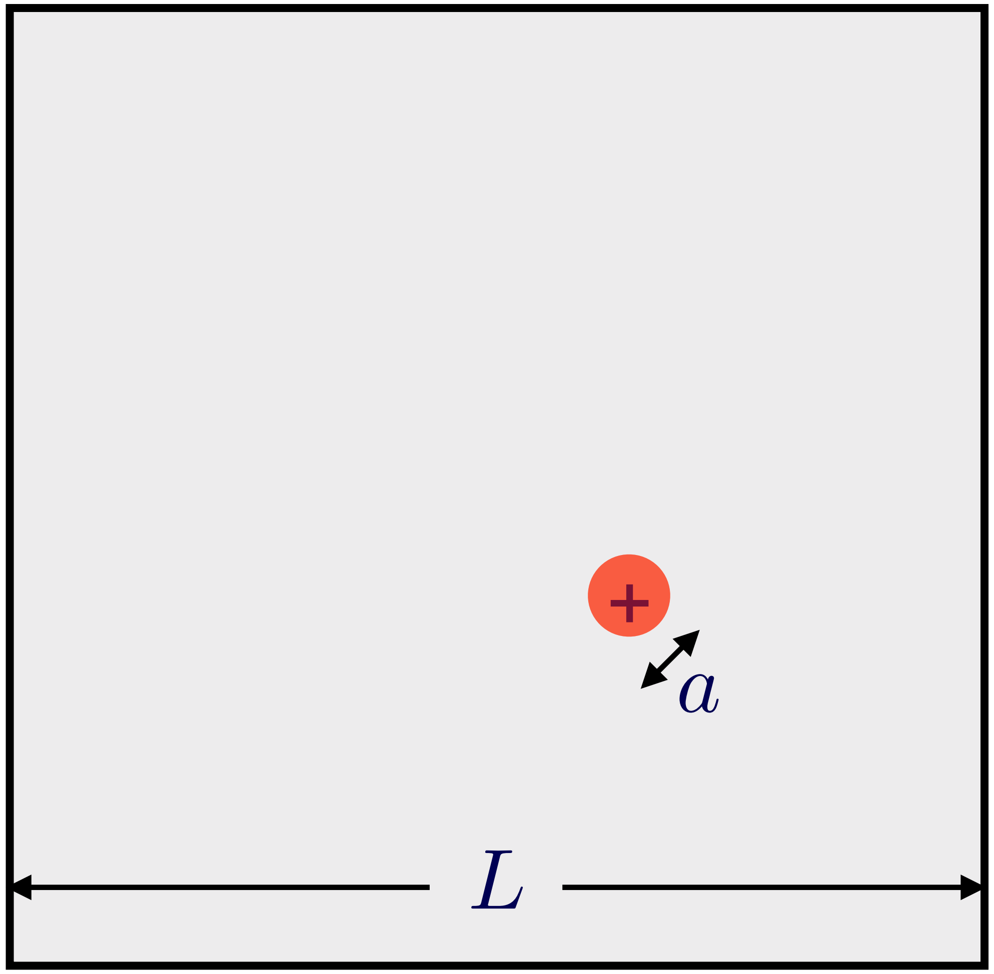

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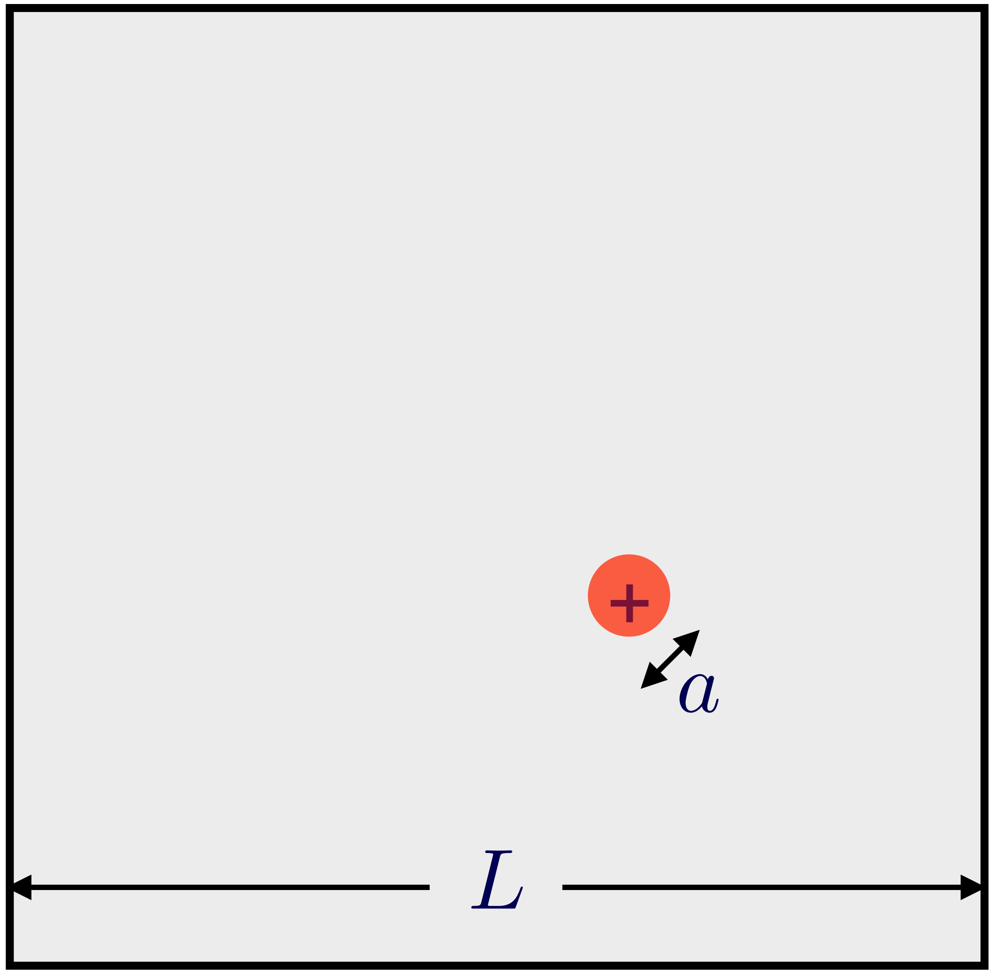


number of configurations $\sim \dfrac{L^2}{a^2}$

$$S_v \sim \ln \frac{L^2}{a^2} \sim \ln \frac{L}{a}$$

Step 3: Vortex Free Energy

Putting all the factors in:

$$F_v = E_v - TS_v = \frac{\pi\hbar^2\rho_s}{m^2}\left(1 - \frac{T}{T_c}\right)\ln\frac{L}{a}$$

Changes sign at

$$T_c = \frac{\pi\hbar^2\rho_s}{2m^2 k_B}$$

Kosterlitz-Thouless Vortex Transition

$$F_v \propto \left(1 - \frac{T}{T_c}\right) \ln \frac{L}{a}$$

$T < T_c$: vortices are expensive and rare (ignoring them is OK)
⇒ phase waves lead to

$$\langle \cos(\varphi_{\mathbf{r}} - \varphi_{\mathbf{0}}) \rangle \propto \left(\frac{a}{r}\right)^{\zeta}$$

$T > T_c$: vortices are cheap and proliferate (can't be ignored) and lead to

$$\langle \cos(\varphi_{\mathbf{r}} - \varphi_{\mathbf{0}}) \rangle \propto e^{-r/\xi}$$

No long-range order in either case, so no "order parameter"

Kosterlitz & Thouless called this a topological phase transition.

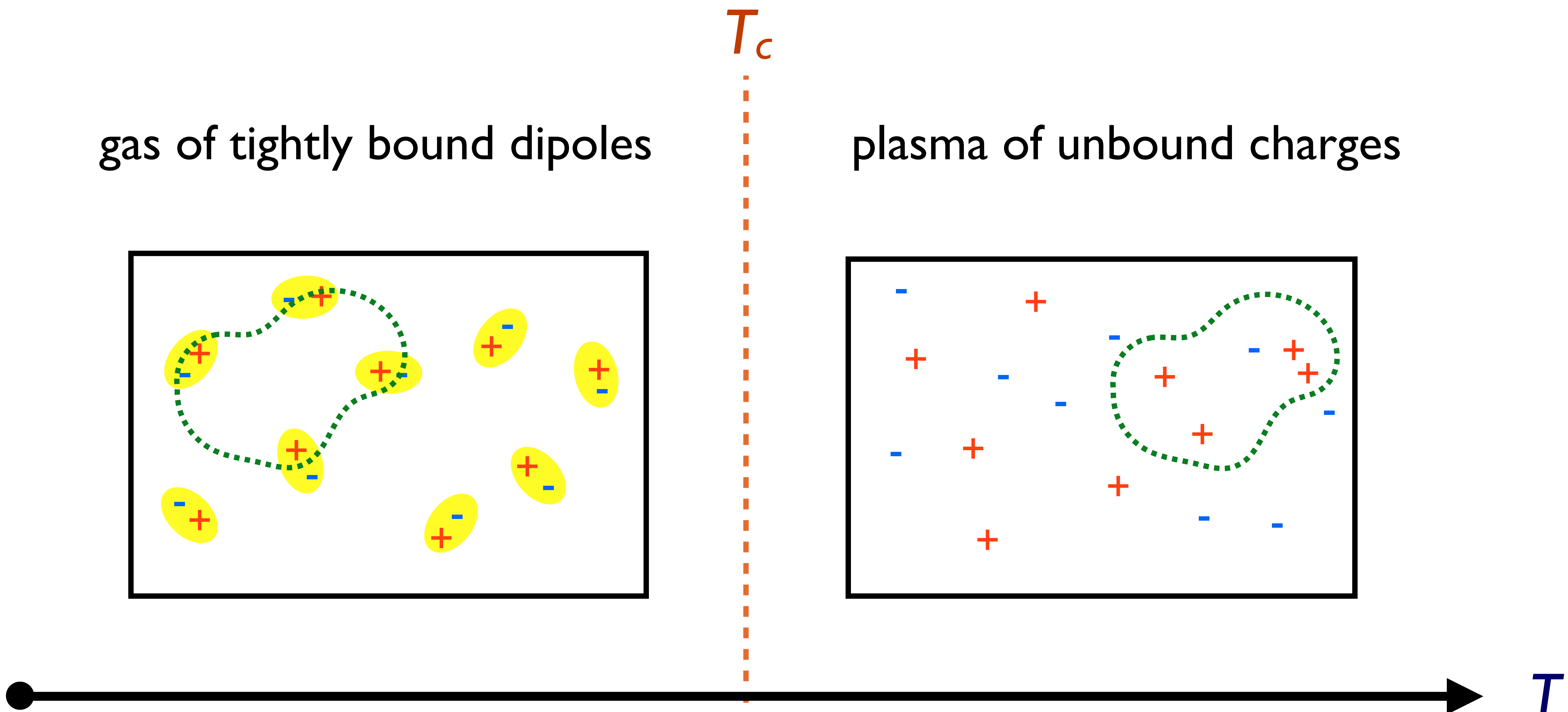
[J.M. Kosterlitz & D.J. Thouless, J. Phys. C **5**, L124 (1973); *ibid*, **6**, 1181 (1973)]

“Coulomb Gas”

Analogy: vortices/antivortices $\longleftrightarrow$ +/- point charges with logarithmic interactions

(useful for *quantitative* calculations)

transition from gas of dipoles to a plasma



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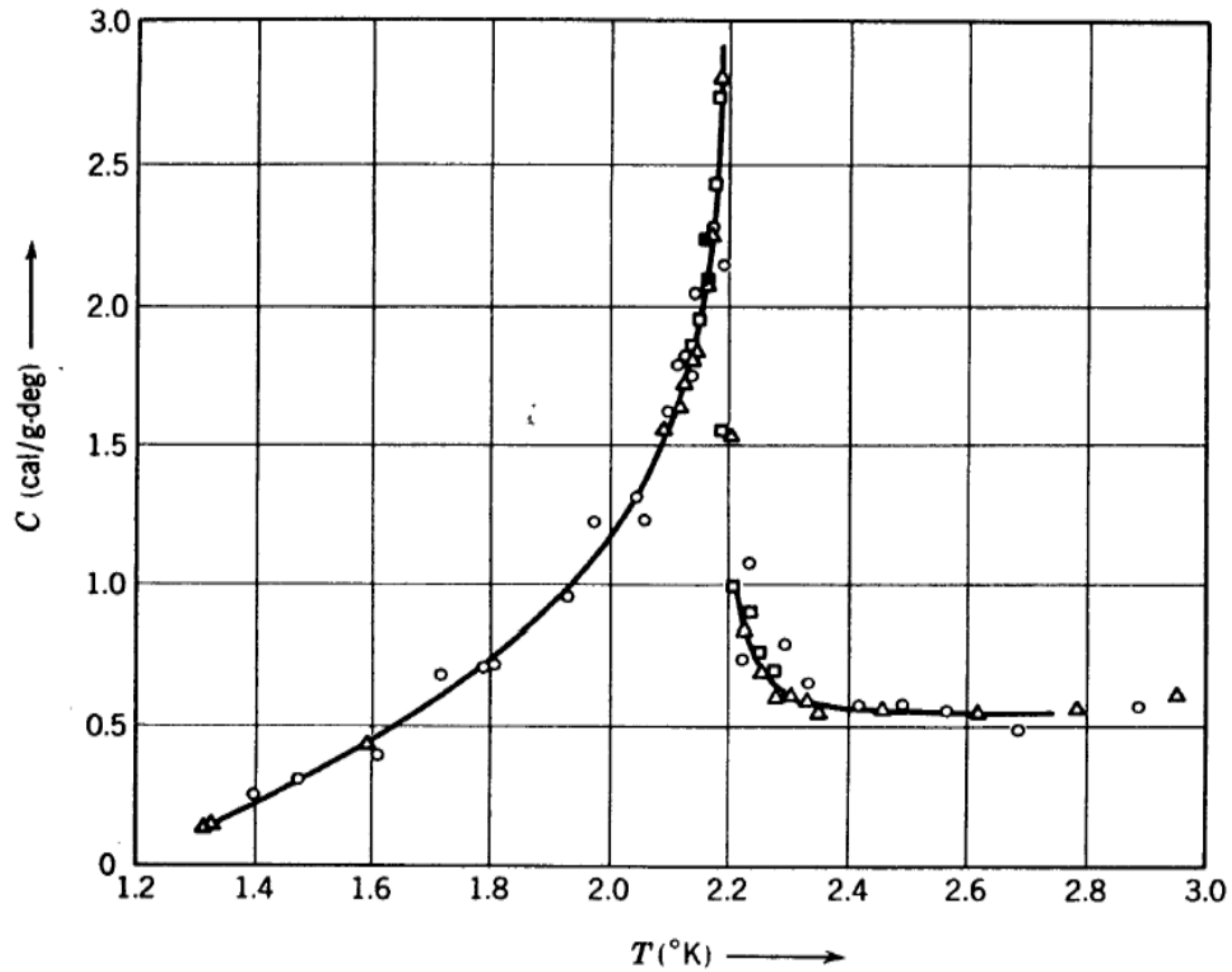
KT Transition: Experimental Signatures

Superfluid transitions in 3D can be seen via thermodynamic measurements

e.g. specific heat



But thermodynamic quantities are smooth across the KT transition!

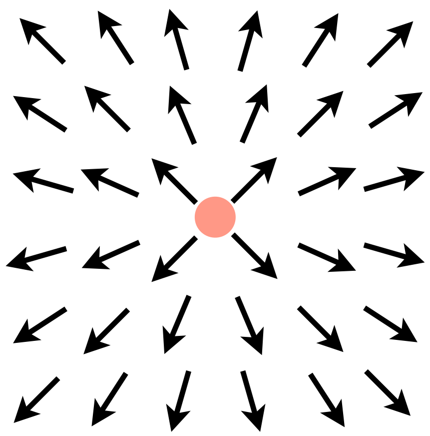
We need a different probe to detect the phase transition.

Image From: F. London, *Superfluids*

KT Transition: Experimental Signatures

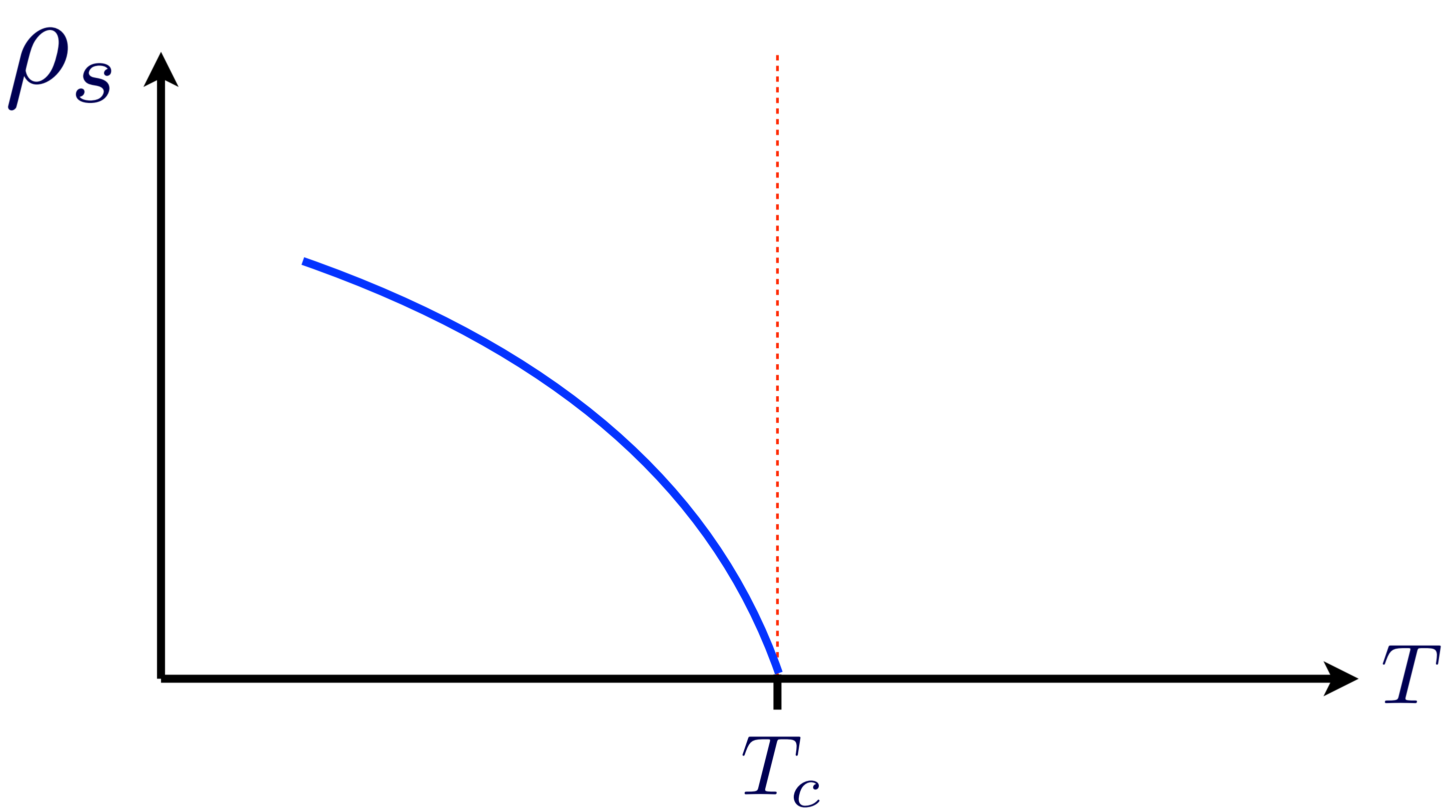
<u>Simplification</u>: assume superfluid density is (roughly) constant everywhere

<u>But</u>: Superfluid wavefunction must **vanish** in vortex core



So as vortices proliferate, superfluid density should <u>decrease</u>

Naïve expectation: ρ_s vanishes continuously at transition



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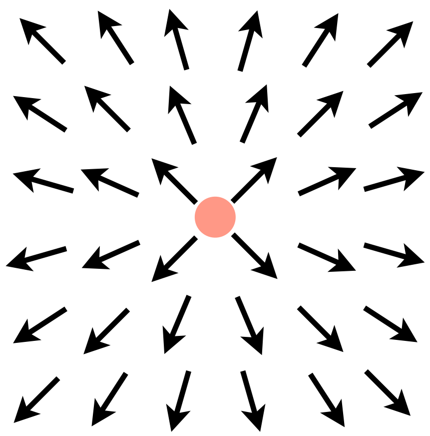

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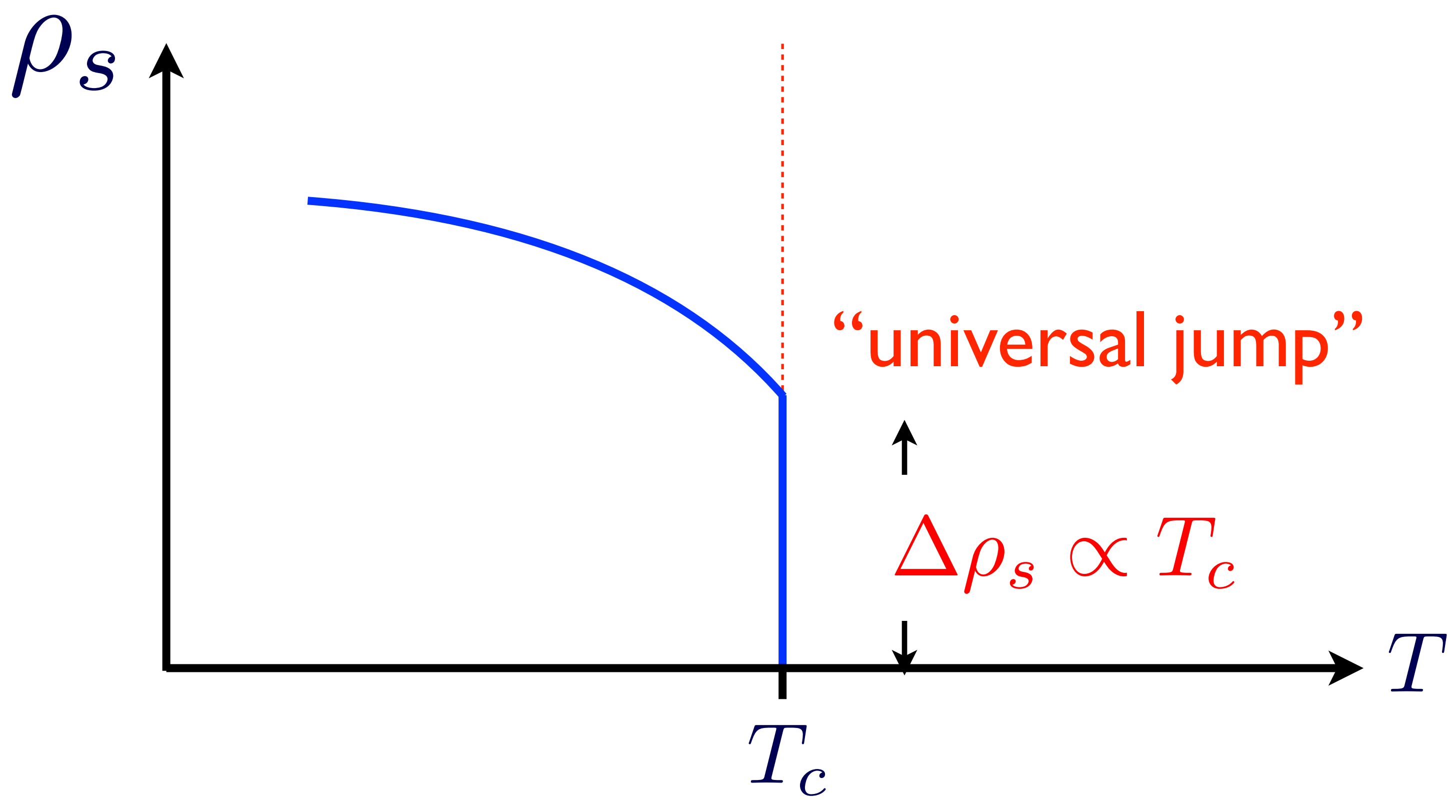
Actually: ρ_s has a "universal jump" at the transition that can be measured



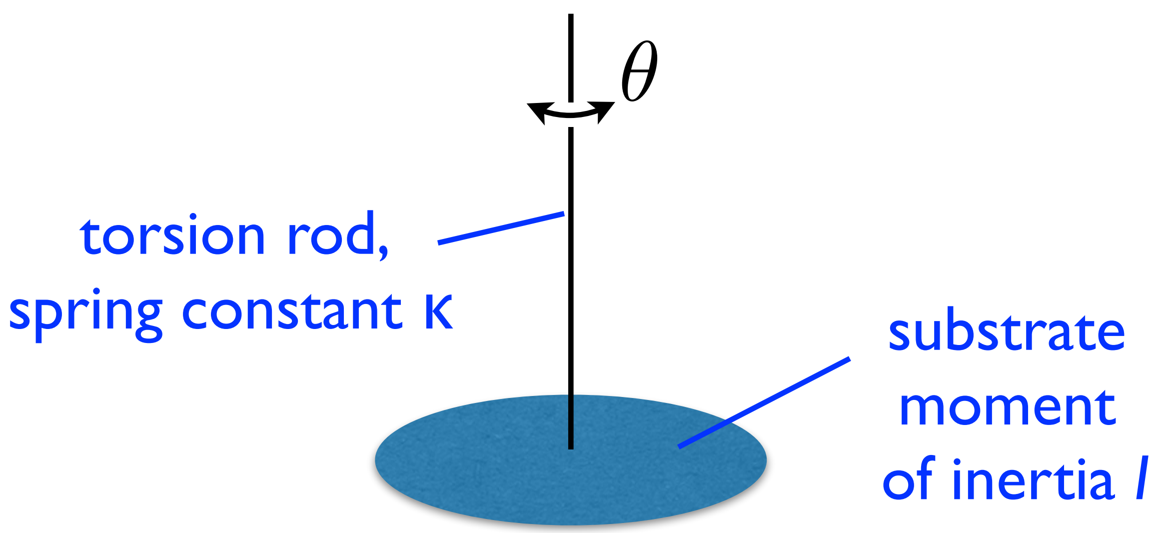
[D. Nelson & J.M. Kosterlitz, PRL **39**, 1205 (1977)]

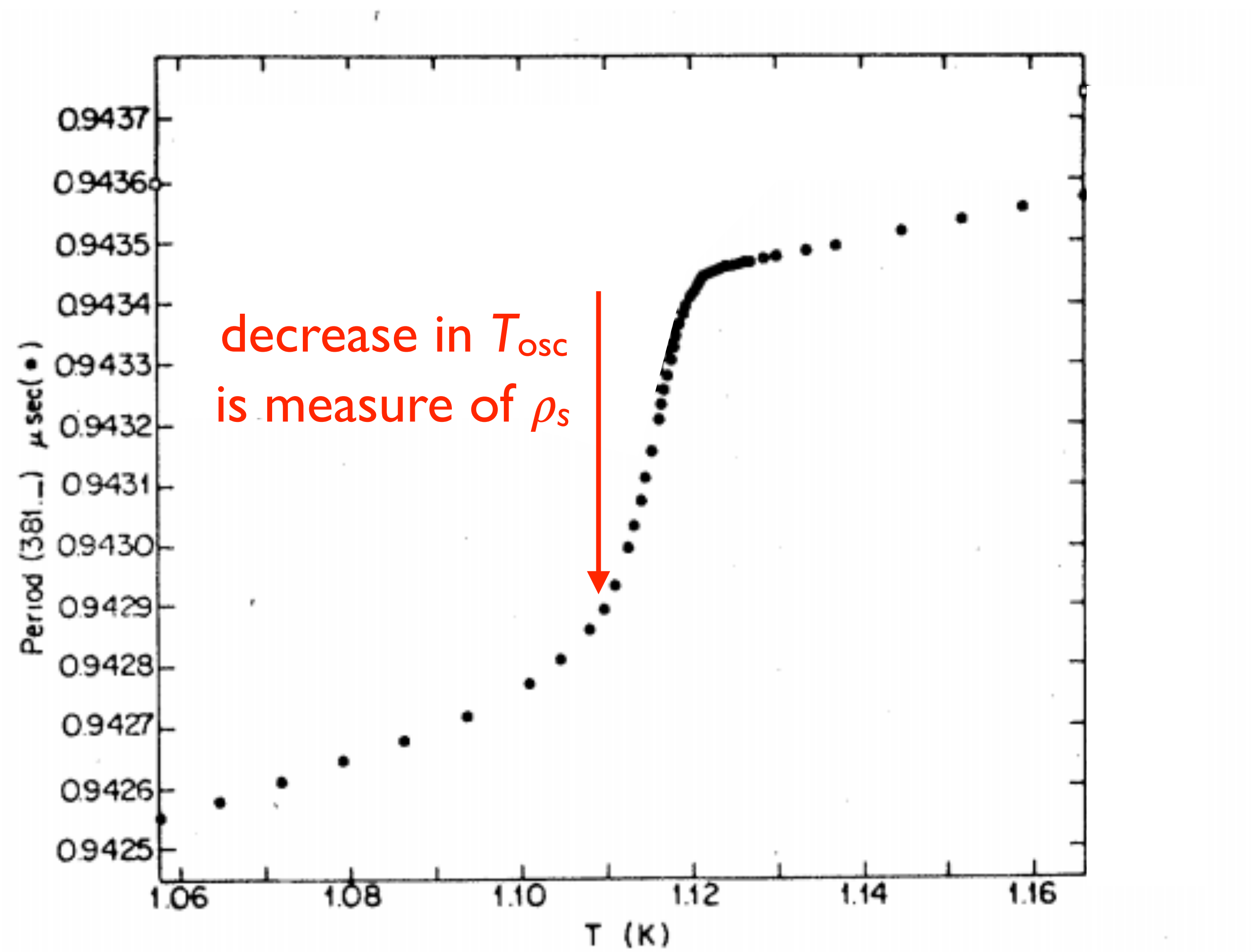
KT Transition: Experimental Signatures

Can measure ρ_s using **torsional oscillator**

$$I\ddot{\theta} = \tau_{\text{restoring}} = -\kappa\theta$$

$$T_{\text{osc}} = 2\pi\sqrt{\frac{I}{\kappa}}$$





$T>T_c$: <u>all</u> the He-4 contributes to I (by sticking to substrate)

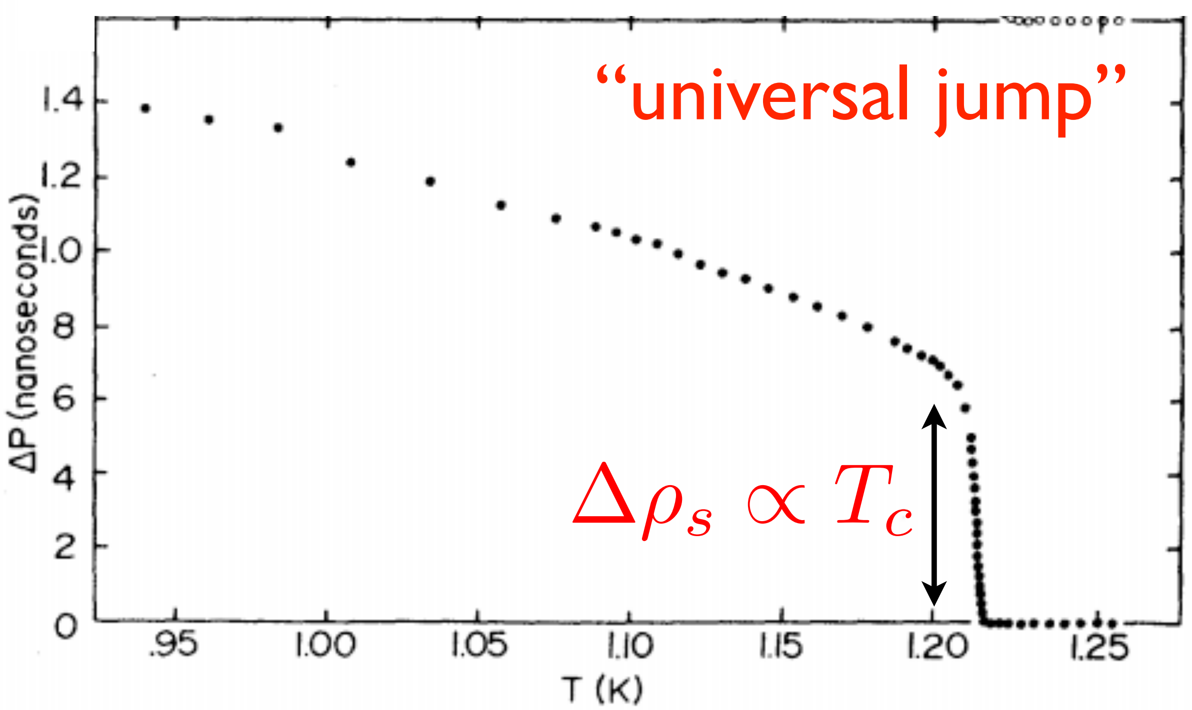
$T<T_c$: superfluid fraction decouples, so I, T_{osc} both decrease

[D.J. Bishop, J.D. Reppy, Phys. Rev. B **22**, 5171 (1980); additional data on dissipation not shown]

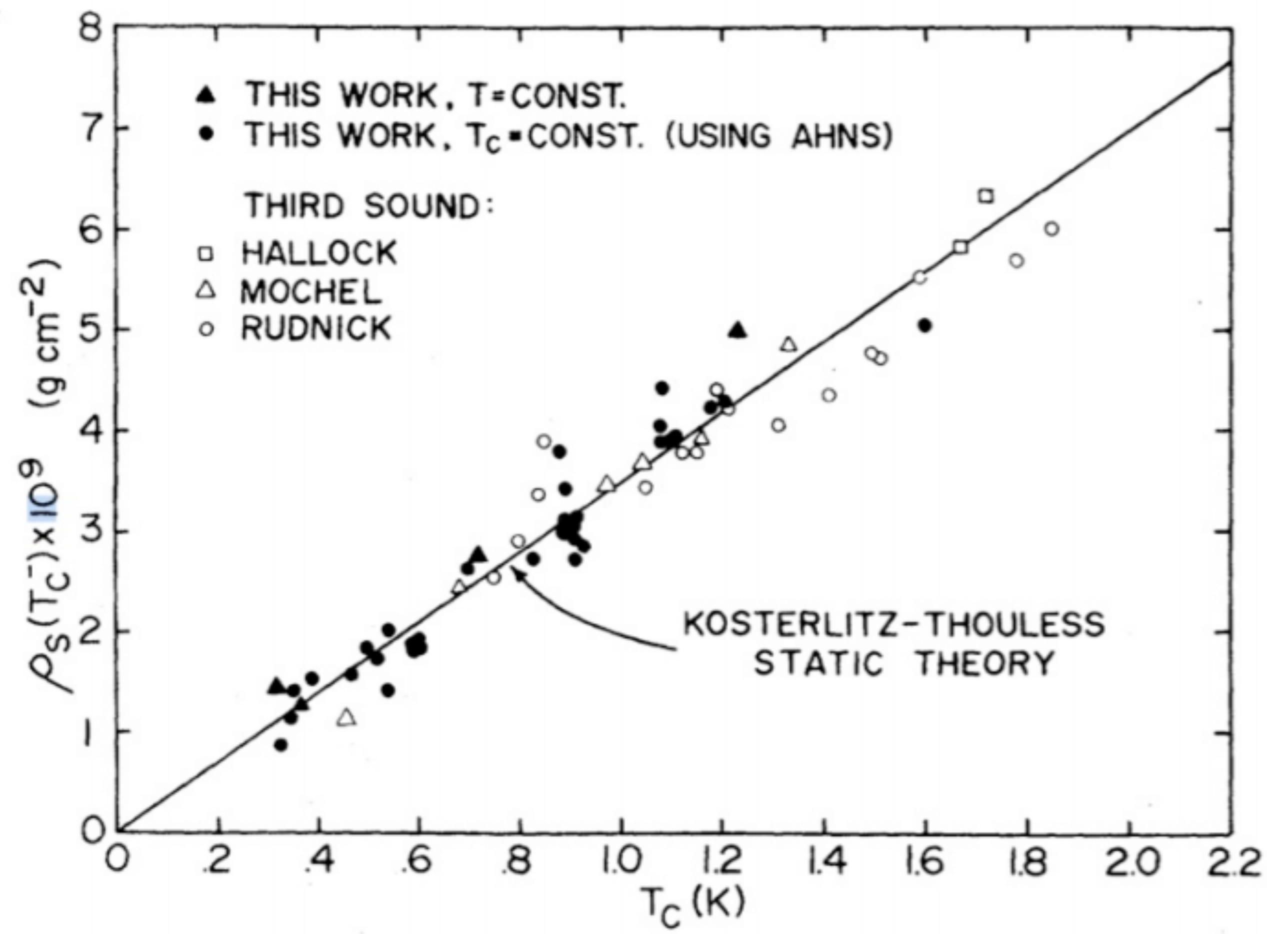
[original expt's: E. L. Andronikashvili, Sov. Phys JETP **18**, 424 (1948), building on ideas of Landau]

KT Transition: Experimental Signatures

Bishop & Reppy: measured ρ_s and found the "universal jump" predicted by the KT theory



[D.J. Bishop, J.D. Reppy, Phys. Rev. Lett. **40**, 1727 (1978)]



(Tune thickness of film to change T_c)

+ also other probes (a.c. response, …)

Honorable Mentions

V.L. Berezinskii

(1935-1980)

- Noted that vortices were important in destroying order (~1.5 yrs before Kosterlitz and Thouless!)
- Understood relation to Coulomb gas + unbinding transition
- **But**: predicted a very different transition, e.g. a <u>continuously</u> vanishing superfluid density
- Died in 1980 after a long illness.

[V.L. Berezinskii, Sov. Phys. JETP **32**, 493 (1971); *ibid,* **34**, 610 (1972)]

F. Wegner

(1940-)

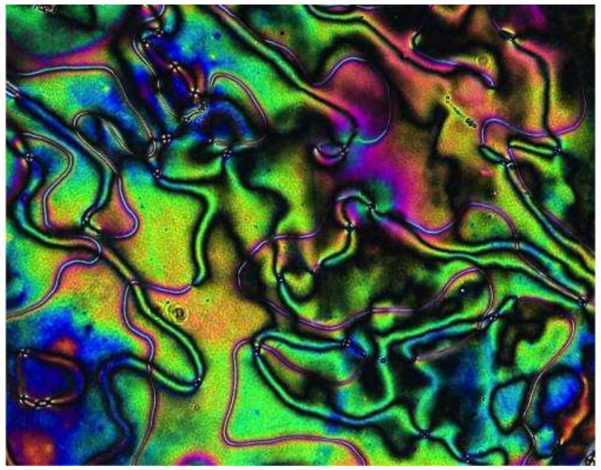
- First model(s) for phase transitions w/o long-range order
- Now understood as confinement-deconfinement transitions in lattice gauge theories
- Crucial to understanding "topological order" and quantum spin liquids

[F. Wegner, J. Math. Phys. **12**, 2259 (1971)]

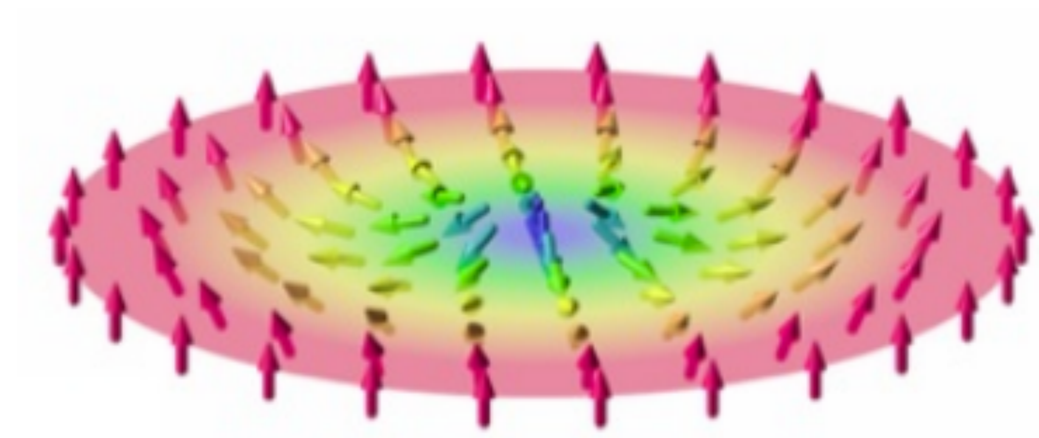
Image Credits: http://www.edu.delfa.net/ ; University of Heidelberg

Variations on a Theme of Kosterlitz and Thouless

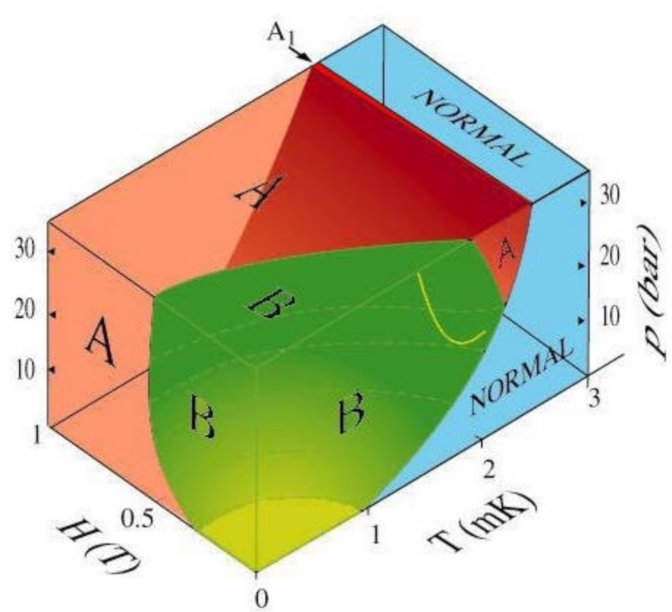
Topological defects are relevant to systems as diverse as:



liquid crystals

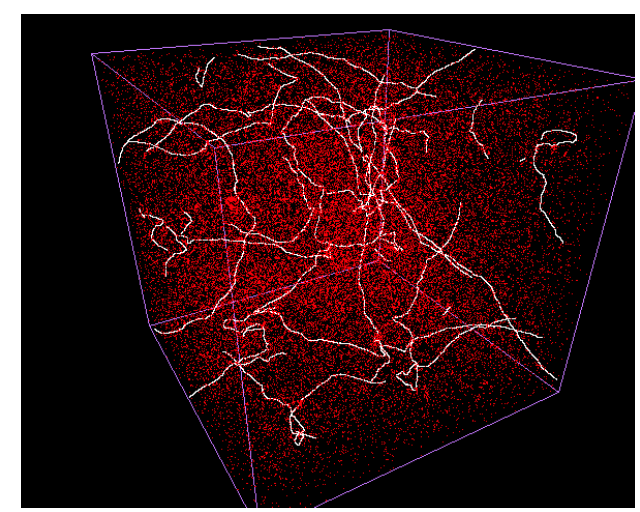


quantum magnets



superfluid ^{3}He

(Tony Leggett's Nobel prize)



"cosmic strings"

in the early Universe

Verduzzo Lab, Rice Univ.; K/ Everschor-Sitte/M. Sitte; http://ltl.tkk.fi/research/theory/he3.html; http://cp3.irmp.ucl.ac.be/~ringeval/strings.html;

Also impact the “big questions” of Condensed Matter

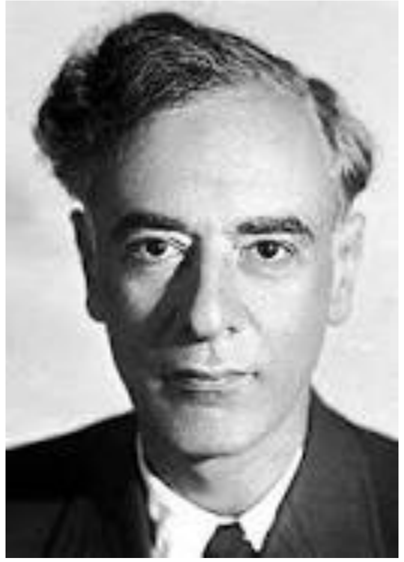
Can we exhaustively classify all phases of matter?

Can we understand all possible phase transitions?

The "Landau Paradigm"

distinct phases of matter = distinct long-range order patterns

phase transitions = changes of these patterns



Lev Landau

crystals

magnets

superfluids

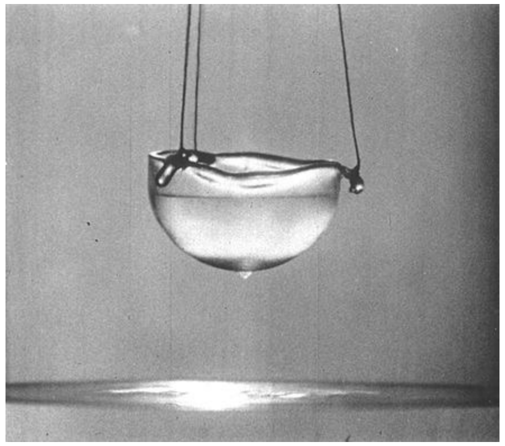


Image Credits: Nobel Foundation; Didier Descouens; Eurico Zimbres ; Alfred Leitner.

Beyond The Landau Paradigm

2D superfluids don't really have long-range order

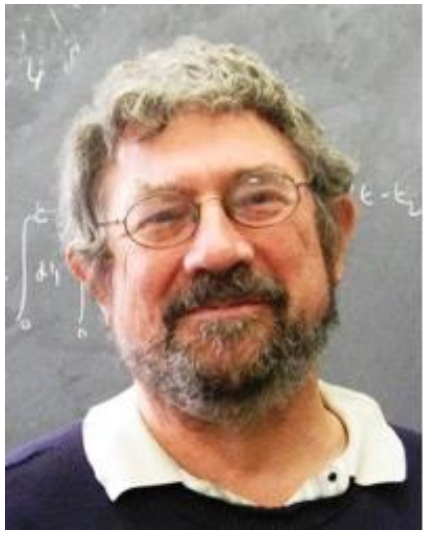
But there **is** a distinct superfluid phase with power-law phase correlations.

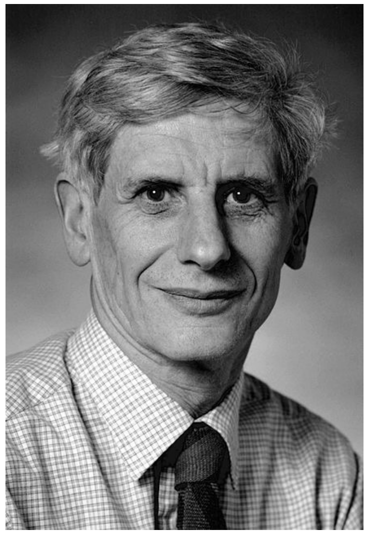
Kosterlitz & Thouless discovered a continuous phase transition that falls outside the Landau paradigm.

In essence, by replacing *symmetry* with *topology* as a guiding principle, we can go beyond the Landau paradigm.

We're still discovering new surprises today.

David J. Thouless, F. Duncan M. Haldane and J. Michael Kosterlitz were awarded the 2016 Nobel Prize in Physics





“for theoretical discoveries of topological phase transitions and topological phases of matter"

They also contributed many other topological ideas to quantum condensed matter — including work that led to the discovery of “topological insulators”.

Image Credits: Brown University; Bengt Nyman; Mary Levin/University of Washington